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Piotr Smarzewski
Lublin University of Technology, Faculty of Civil Engineering and Architecture, ul. Nadbystrzycka 40, 20-618 Lublin, Poland
Corresponding author. E-mail: p.smarzewski@pollub.pl

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THE ANALYSIS OF LOAD CAPACITY AND DEFLECTION OF REINFORCED HIGH STRENGTH CONCRETE BEAM

The article presents the analysis of the load capacity and deflection of a reinforced high strength concrete beam with a low level of reinforcement in the process of static deformation in comparison to experimental data, according to the methodology of determining the parameters for the constitutive model of high strength concrete, based on its ultimate uniaxial compressive strength. Three variants of numerical solutions of the BP-1a beam were presented, which are distinguished by their failure surface. Numerical calculations made with use of the Newton-Raphson method with adaptive descent and the arc length method are verified with test results. The load-deflection relationships for the flexure beam from the experimental data and the finite element analyses are in good agreement up to failure. In all the cases, the differences between the final loads for the model and the ultimate loads for the experimental results are less than 5%.

Keywords: finite element method, reinforced concrete, beam, high strength concrete

ANALIZA STANU NOŚNOŚCI I PRZEMIESZCZENIA BELKI ŻELBETOWEJ Z BETONU WYSOKIEJ WYTRZYMAŁOŚCI

W artykule przedstawiono analizę stanu nośności i przemieszczenia belki żelbetowej o niskim stopniu zbrojenia wykonanej z betonu wysokiej wytrzymałości w procesie statycznego odkształcania w porównaniu z wynikami eksperymentalnymi według metodyki ustalania parametrów modelu konstytutywnego betonu wysokiej wytrzymałości na podstawie jego wytrzymałości na ściskanie. Przedstawiono trzy warianty rozwiązania numerycznego belki BP-1a. Zróżnicowano je przez powierzchnię graniczną betonu. Obliczenia numeryczne wykonane metodą Newtona-Raphsona ze spadkiem adaptacyjnym i metodą długości łuku zweryfikowano wynikami doświadczalnymi. Zależność obciążenie--przemieszczenie uzyskana doświadczalnie dla belki zginanej jest zgodna aż do zniszczenia z analizami metodą elementów skończonych. We wszystkich przypadkach różnice pomiędzy końcowymi obciążeniami dla modeli i doświadczalnymi wynikami są mniejsze niż 5%.

Słowa kluczowe: metoda elementów skończonych, żelbet, belka, beton wysokiej wytrzymałości

INTRODUCTION

Computer simulations for real systems are most frequently done with linear models due to their numerical simplicity and easy interpretation. Such a premise is valid only for precisely defined conditions of the system, e.g. lack of large deformations, in the phases before material plastification or lack of contact. In all other situations it is necessary to do nonlinear calculations in order to map the real situation. The possibility to design any three dimensional model and consider its nonlinearity generates a new effective approach to analyses made by personal computers. The article discusses the results of the analyses of load capacity and deformations of a model high strength reinforced concrete beam in the process of static deformation.

PARAMETERS OF HIGH STRENGTH CONCRETE MODEL

In the numerical model of a spatial high strength reinforced concrete beam, the dimensions equal to a rectangular BP-1a beam were used [1]. In order to shorten the computing time, half of the element was modelled. The boundary conditions were modelled in such a manner as to obtain the highest possible accuracy with the experimental beam. The boundary conditions in the symmetry plain were used. Figure 1 presents the boundary conditions in the analysed model.

The results of the numerical calculations made by the Newton-Raphson method with adaptive descent [1] and the definition of Crisfield's arc length [2] were compared to the experimental data [1]. Three variants of solutions of the beam and the parameters of the material models were presented in the work [4]. All of the proposed solutions of the concrete model for the BP-1a beam were described with the failure surface by William-Warnke [5] and its evolution based on numerical analysis and the result verification for real reinforced concrete beams [6].

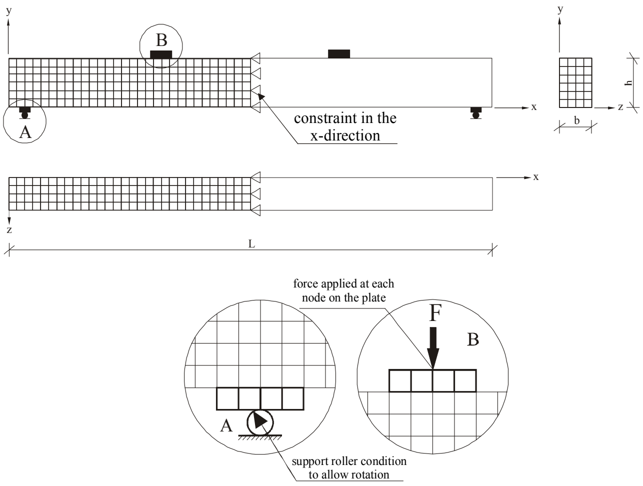

Fig. 1. Boundary conditions for planes of symmetry at support element and load application location for ½ BP-1a beam

Rys. 1. Warunki brzegowe w płaszczyźnie symetrii, na podporze i w strefie przyłożenia obciążenia ½ belki BP-1a

The construction properties of the BP-1a beam are characterised by the following parameters of constitutive models: for high strength concrete - uniaxial compressive strength f_c = 81.2 MPa, modulus of elasticity E_c = 36817 MPa, uniaxial tension strength f_t = 4.87 MPa, Poisson's ratio ν_c = 0.15, density ρ_c = 2600 kg/m^3, compressive strain at the peak stress $f_c - \varepsilon_{c1}$ = 6 ‰, ultimate compressive strain ε_{cu} = = 12 ‰, shear transfer coefficients for an open crack β_t = 0.5 and shear transfer coefficients for a closed crack β_c = 0.99; for reinforced steel, the steel support and steel loading plate: modulus of elasticity E_s = 200 GPa, yield stress for ϕ10 rods made of A-III steel f_y = 410 MPa, for ϕ6 rods made of A-II steel f_y = 355 MPa, Poisson's ratio ν_s = 0.3, density ρ_s = = 7800 kg/m^3. The properties of the construction material of BP-1a beam are defined by additional parameters describing the five-parameter failure surface of high strength concrete: ultimate biaxial compressive strength f_{cb} = 97.4 MPa, ultimate compressive strength for a state of biaxial compression superimposed on a hydrostatic stress state $\sigma_h^a - f_1$ = 117.7 MPa, ultimate compressive strength for a state of uniaxial compression superimposed on a hydrostatic stress state $\sigma_h^a - f_2$ = = 140.1 MPa, ambient hydrostatic stress state describing the average distribution of normal stresses, σ_h = 140.5 MPa and multiplier for the amount of tensile stress relaxation T_c = 1.

LOAD INCREASE AND FAILURE INTERPRETATION

The load increase in the numerical analyses of reinforced concrete constructions is defined by the experimental curve load-deflection. Figure 2 presents the diagram of vertical deflection at the midspan of the lower beam boundary from the summary load, with consideration for distinctive phases of the work of reinforced concrete construction.

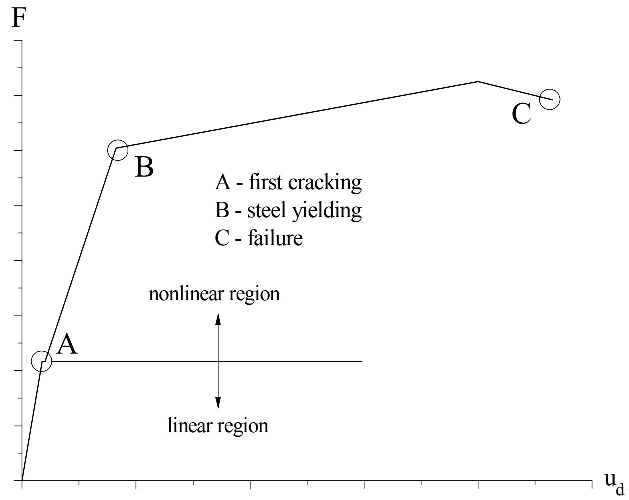

Fig. 2. Diagram of load-vertical deflection at midspan for beam from summary load

Rys. 2. Schemat przemieszczenia pionowego w punkcie środkowym na dolnej krawędzi belki od obciążenia sumarycznego

In the range of linear elastic, the model is stable and it is not necessary to use many steps of load increments $(\Delta F_{0A} = 1)$. In the zone of cracking formation (A), the load is gradually applied with small increments equal to $\Delta F_A = 0.001$, as at this stage of analysis, obtaining convergent results is relatively difficult. In the zone of non-elastic strains, after the first cracking, the analysis is again fast-convergent $(\Delta F_{AB} = 0.01)$. In the reinforcing steel yielding zone (B) the method is more slowly convergent and the convergence of the solution is ensured by maintaining the smallest possible step of load increments, equal to $\Delta F_B = 0.0005$. At the final stage of numerical solution, after steel yielding, the load increment causes the creation of a significant number of cracks. In this range, the minimum increment of load $\Delta F_B = 0.0005$ was used. It was assumed in each model that the failure (C), takes place at the moment of solution divergence at a minimum increment of load $\Delta F_C = 0.0005$.

ANALYSIS OF LOAD CAPACITY AND DEFLECTION

The course of load-deflection changes of the central point at the lower boundary of the beam at midspan that can be observed in Figure 3 is non-monotonous and coherent with the model of beam behaviour with material damage in the process of deflection and the use of numerical methods that generate complete solution paths with local descent of stiffness and global construction softening. The research on high strength reinforced concrete beams by Rashid and Mansur [7] proves that the effects of beam failures in the tensing zone are not compensated by the elastic properties of steel and plasticity of concrete in the compression zone. Therefore, the effects of softening are generated, which can be seen in the curve of the relation between deflection and load in the form of immediate falls of load.

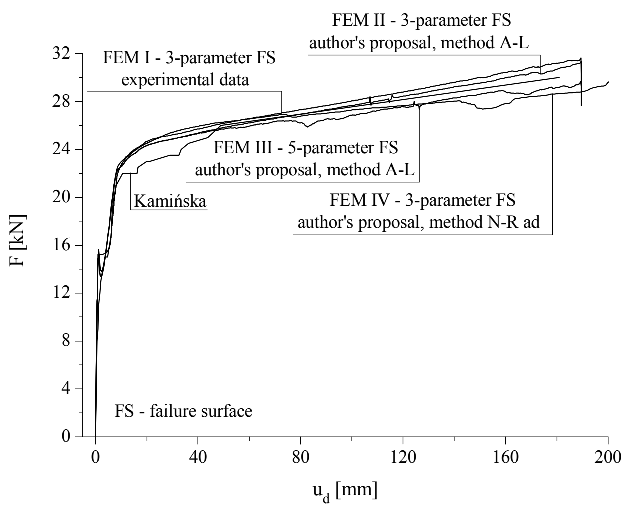

Fig. 3. Comparison of load-vertical deflection at midspan of BP-1a beam and load, calculated by different numerical methods

Rys. 3. Zestawienie zależności przemieszczenia pionowego w środku belki BP-1a od obciążenia uzyskanych różnymi metodami numerycznymi

In the zone of elastic stresses and after the cracking, the model beam is characterised by a slightly higher stiffness, as in the model matrix material, the microcracks and the difference in the concrete module at compressing and tensing were ignored. The yielding phase of the reinforcing bars is presented in the diagram of load-deflection by an immediate decrease in beam stiffness. The ultimate limit deflections at the beam midspan in the inside diameter of supports of 3 m, registered in the numerical solutions were 189.6 mm, 189.6 mm and 199.9 mm respectively. The value of ultimate limit deflection registered in the experiment was 180.9 mm.

In the numerical solutions, the ultimate limit state was recorded for the loads of F_u = 31.3 kN, F_u = = 29.7 kN and F_u = 31.8 kN. Although in Kamińska's [1] experiment, the crush of the beam was not observed as it was prevented by extensive deflections of the beam that reached 20 mm at midspan, which exhausted the technical capacity of the test bench, the value of maximum load F_u = 30 kN, calculated from the deflection conditions, is coherent with the value of failure load obtained from all the numerical solutions. Figure 4 presents the distribution of load-vertical deflection u_d at the midspan of the lower boundary for different levels of load. In both numerical solutions obtained by Crisfield's arc length method, similar values of vertical load-deflection u_d were obtained. Therefore, the variations of lower boundary deflection were presented only for the first variant of the numerical solution.

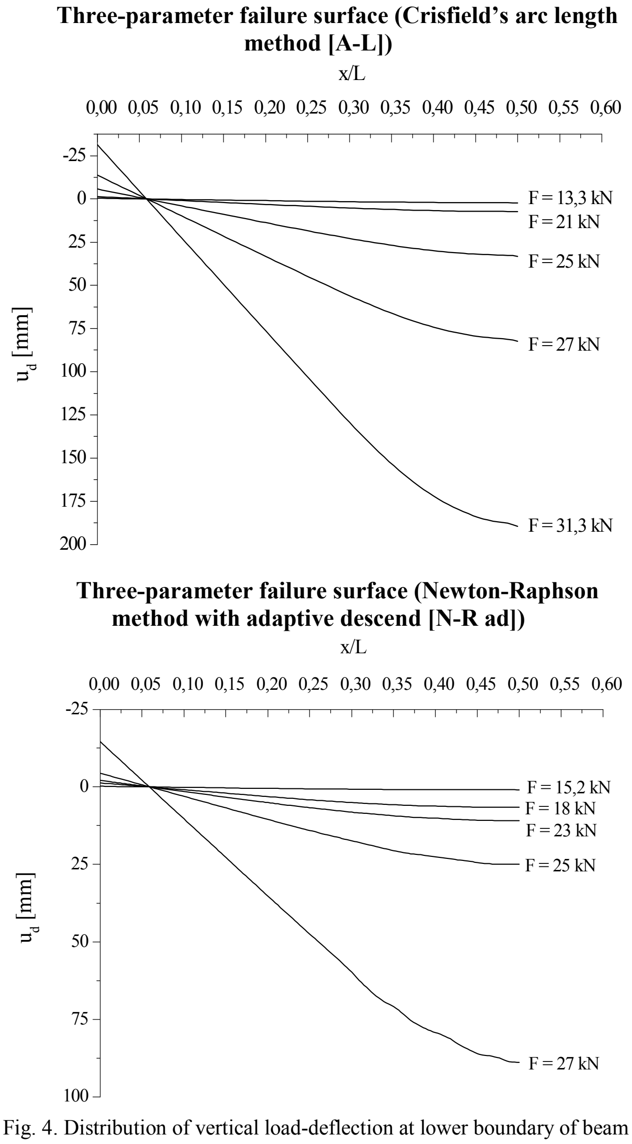

Fig. 4. Distribution of vertical load-deflection at lower boundary of beam

Rys. 4. Rozkłady przemieszczeń pionowych dolnej krawędzi belki

Comparing the relations of the vertical deflection for the points at the midspan of the lower - u_d and upper u_g boundary in the function of load - F, one may notice a correlation that vertical deflections at the lower boundary are approximately equal to the deflections at the upper boundary of the element.

SUMMARY

The use of the incremental-iterative method of Crisfield's arc length allows one to obtain a complete path of load-deflection with both local and global construction softening. Moreover, the algorithm is characterised by high efficiency; a variable step of load increments and properly selected arc lengths guarantee shortening the time of numerical computing, still with very precise solutions. The usefulness of the arc length method was verified on a spatial model of a reinforced concrete beam with consideration for the deformative softening of concrete during compressing or tensing. The method can be particularly attractive for construction engineers in the situation when the precise definition of deflection for a given load is necessary.

The numerical solutions obtained for the reinforced concrete beam are coherent with the results obtained for a real element. The results supported by measurements prove that the brittleness of high strength concrete in the bending beams is limited due to good joint action between the concrete and reinforcing steel. Clear correlation of the experimental and numerical results depends on proper selection of the precise linear and non-linear material properties.

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