

Rafał Chatys*, Krzysztof Piernik

Kielce University of Technology, al. 1000-lecia Państwa Polskiego 7, 25-314 Kielce, Poland

*Corresponding author. E-mail: chatys@tu.kielce.pl

Received (Otrzymano) 01.08.2016

NUMERICAL ANALYSIS OF DESIGN OF FIBER COMPOSITE MATERIALS WITH POLYMER MATRIX

Verification of the correct modeling of the flat state (on the example of a sample) was carried out using ABAQUS finite element analysis, by entering in the program, the mean values of samples cut at different angles to the reinforcement (mato E-glass fabric). Then, from the same material a tubular element subjected to special loads, made possible thanks to a rigid element (in the form of rods) so-called Rigid Links, was modeled.

Keywords: composite, strength, Rigid Links, ABAQUS

NUMERYCZNA ANALIZA PROJEKTOWANIA WŁÓKNISTYCH MATERIAŁÓW KOMPOZYTOWYCH O OSNOWIE POLIMEROWEJ

Weryfikację poprawnego zamodelowania stanu płaskiego (na przykładzie próbki) przeprowadzono metodą analizy elementów skończonych ABAQUS, wprowadzając do programu średnie wartości z próbek ciętych pod różnymi kątami względem wzmocnienia (mato tkanina szklana typu E). Następnie z tego samego materiału został zamodelowany element rurowy poddany obciążeniom specjalnym możliwym dzięki sztywnemu elementowi (w postaci prętów), tzw. Rigid Links.

Słowa kluczowe: kompozyt, wytrzymałość, sztywny element, ABAQUS

INTRODUCTION

Intensive development of the production of fibrous composite materials (FCM) is the basis for the design of complex elements or structural components. Models showing the behavior of the polymer material cannot always be described clearly, despite their great diversity, and in some cases, complex construction [1]. For this reason, rheological models of viscoelasticity are developed which determine the non-linear stress-strain relations [2, 3]. Then, the most important factors are determined by basic research, taking into account the specific properties of the components of the formed laminated composite [4].

An additional aspect, besides appropriate selection of the components and methods of forming composite sandwich panels, is the quality and maintaining the technological parameters [5]. The mechanical properties of polymer laminates in terms of the linear theory of elasticity are described by physical equations for anisotropic centers resulting from the generalized Hooke's law:

$$\varepsilon_{ij} = S_{ijkl}\sigma_{kl} \quad (1)$$

$$\sigma_{ij} = Q_{ijkl}\varepsilon_{kl} \quad (2)$$

for $i, j, k, l = 1, 2, 3$.

The fourth-order tensors have 81 components. However, after taking into account material symmetry, Equations (1) and (2) can be reduced to the form in which the tensors have only 36 components (of which 21 are independent components).

For example, the stress and strain tensors also reduce from 9 to 6 independent components [6, 7], taking into account the symmetry of the tensors. Then Equation (2) is in the form of a matrix:

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & Q_{13} & Q_{14} & Q_{15} & Q_{16} \\ Q_{21} & Q_{22} & Q_{23} & Q_{24} & Q_{25} & Q_{26} \\ Q_{31} & Q_{32} & Q_{33} & Q_{34} & Q_{35} & Q_{36} \\ Q_{41} & Q_{42} & Q_{43} & Q_{44} & Q_{45} & Q_{46} \\ Q_{51} & Q_{52} & Q_{53} & Q_{54} & Q_{55} & Q_{56} \\ Q_{61} & Q_{62} & Q_{63} & Q_{64} & Q_{65} & Q_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 \end{bmatrix} \quad (3)$$

Estimating, or modeling the properties of polymer composites using microstructural means (based on torque viscoelasticity theory, that is physico-mathematical foundations), is in the process of being created (although the initial works date back to the end of the 1960 s [3]). In practice it is often possible to encounter the situation when the matrix of a laminate is a mixture

of several polymers with different physical and chemical properties, and the intersecting network of these polymers are distinct phases. Takayanagi models, i.e. modifications of Maxwell, Kelvin-Voigt and Zener models, enable the description of such materials by introducing coefficients taking into account the volume contribution of the reinforcement (V_λ) and matrix (V_ϕ) of the polymer mixture [7]. The main task of the engineer is to evaluate the ability of the structure to withstand loads, which is not possible without knowing the strength characteristics of the material used in the object in question. They can be determined experimentally or on the basis of analysis of the internal structure of the material [6].

ANALYSIS OF STRESS STATE IN FLAT AND CYLINDRICAL ELEMENTS

An approximate solution to torsion issues of tubes (Fig. 1) was carried out using an analogy membrane in which sections AB and CD respectively represent the level of the outer and inner edges, while AC and BD are cross sections of the membrane extending between these edges (in the case of thin tube walls we can skip changing the membrane thickness and assume that the AC and BD are straight lines).

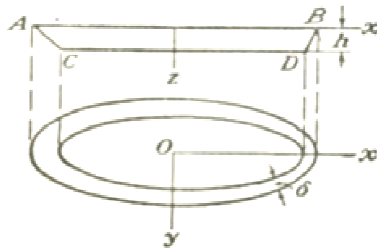


Fig. 1. Diagram thin-walled tubes with markings [8]
Rys. 1. Schemat z oznaczeniami rury cienkościennej [8]

With the above assumption, the tangential stresses in the thin wall thickness are distributed evenly. Then, by designating the difference in the levels of the two edges and the variable thickness by h , the tangential stress at any point in the wall is defined by the angle of the membrane (4), which is inversely proportional to the wall thickness (they are therefore the greatest where the wall thickness is the smallest [8, 9]).

$$\tau = \frac{h}{\delta} \tag{4}$$

In order to determine the relationship between the stresses and the torque, an analogous membrane was used to calculate the torque from the volume of ACDB.

Then, the torque takes the form:

$$M_t = 2Ah = 2A\delta\tau \tag{5}$$

for: A - is the mean surface area of the tube cross-section.

Then from Equation (5) to determine the tangential stress we receive the expression:

$$\tau = \frac{M_t}{2A\delta} \tag{6}$$

In order to determine the torsional angle, we use the equation:

$$\int S \left(\tau \frac{q}{s} \frac{1}{2G\theta} \right) ds = qA \Rightarrow \tau ds = \frac{M_t}{2A} \int \frac{ds}{\delta} = 2G\theta A \tag{7}$$

The angle of rotation can be expressed as:

$$\theta = \frac{M_t}{4A^2G} \int \frac{ds}{\delta} \tag{8}$$

After further transformation we will receive

$$\theta = \frac{M_t s}{4A^2G\delta} \tag{9}$$

where s is the length of the center line of the tube section [8].

The FEM concept assumes that each value (e.g., displacement, stress) is described by a continuous function (primary) in a given area (a continuous fragment of a physical model), which approximates a discrete model (i.e., a set of continuous functions defined in a finite number of subdivisions called finite elements to which the area under consideration is divided [10, 11]). The aim of the work is to use the capabilities of a finite element program (which ABAQUS belongs to), to estimate the strength properties of composite material produced by one of the methods of resin injection molding, under pressure into the mold cavity. The assumptions about the loads and fields of internal forces in the shells (using ABAQUS shell finite elements - Fig. 2) are similar to those of plates (except for the fact that in shells curved coordinates $\{\xi_1, \xi_2, \xi_3\}$, are used, ξ_1, ξ_2 of which, are connected with the central surface and ξ_3 is perpendicular to that surface [12]).

With this in mind, we will have the relationship:

$$\sigma_{ij}(\xi_1, \xi_2, \xi_3) = \sum_k F^{(k)}(\xi_1, \xi_2) [g_{ij}(\xi_3)]^{(k)} \tag{10}$$

where the functions $[g_{ij}(\xi_3)]^{(k)}$ come down to a 2D object.

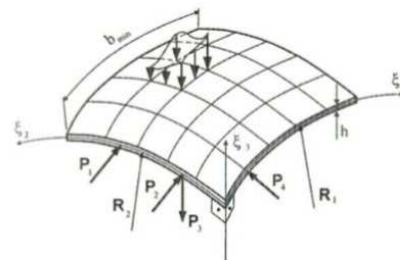


Fig. 2. Determining coordinates in shell [12]
Rys. 2. Określenie współrzędnych w powłoce [12]

The stress was determined with the help of the most frequently used energy hypothesis - the von Mises

hypothesis [10, 13], which as a measure of strain takes the specific energy of the non-dilatational strain. Dependencies (11) and (12) show respectively the reduced stress (von Mises hypothesis) in the principal stress system and for the flat stress state $\sigma_x, \sigma_y, \tau_{xy}$.

$$\sigma_{red} = \frac{1}{\sqrt{2}} \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2} \quad (11)$$

$$\sigma_{red} = \sqrt{[(\sigma_x - \sigma_y)]^2 + 3\tau^2} \quad (12)$$

if $\sigma_x = \sigma, \sigma_y = 0, \tau_{xy} = \tau$ then the above dependency (11) assumes the form [12]:

$$\sigma_{red} = \sqrt{\sigma^2 + 3\tau^2} \quad (13)$$

The projection of this hypothesis (the greatest tangential stresses as a hexagon - dashed line) in the principal stress system is a circular cylinder equally inclined to the axis (Fig. 3a), and the trace of this cylinder in a flat stress system is an ellipse (solid line Fig. 3b)

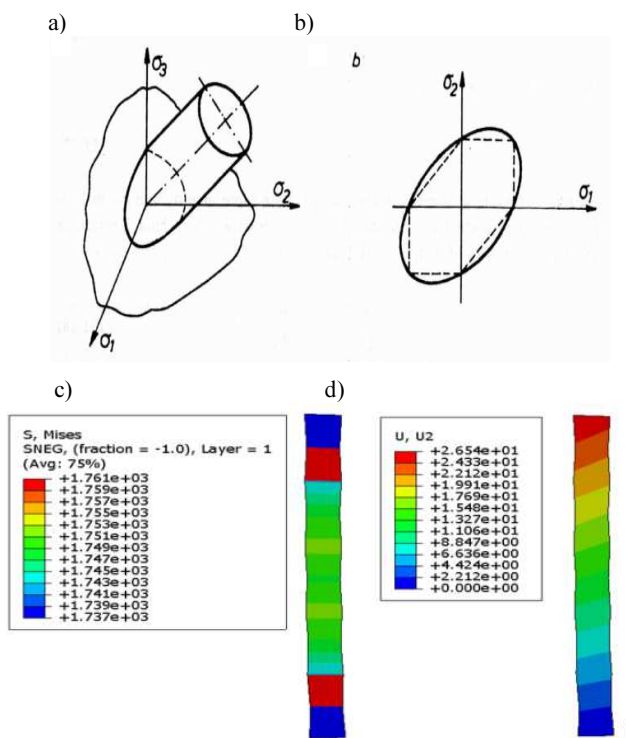


Fig. 3. Projection of Huber hypothesis in main screen (a) and flat (b) system stress with modeled stress dependency (c) and displacement (d) of sample [14]

Rys. 3. Odzworowanie hipotezy Hubera w głównym (a) i w płaskim (b) układzie naprężeń z zamodelowaną zależnością naprężenia (c) i przemieszczenia (d) w próbce płaskiej [14]

EXECUTION OF MODELS

In order to apply the central load in the finite element method, infinitely rigid elements of the so-called Rigid Links were used. They can be used to model adhesive joints or welds as they may have a much greater strength than the native material. In the case of items with complex shapes that carry significant concentrated

loads, it is difficult to predict where they will be the most overloaded. Therefore, in the cross-section of such an element, e.g. a tube, several rigid links are formed, and at their central point the concentrated force is applied [15, 16].

An additional advantage of rigid links is the possibility of releasing particular degrees of freedom and rotation angles [9, 16], allowing one to predict the behavior of the material during loads changes in the construction or breaking of any actual connection. This function is particularly useful in the case of load simulation of composite materials, carrying central load in the form of a concentrated force. Because the variety and number of stacked layers, composite materials behave differently than metals when carrying such loads.

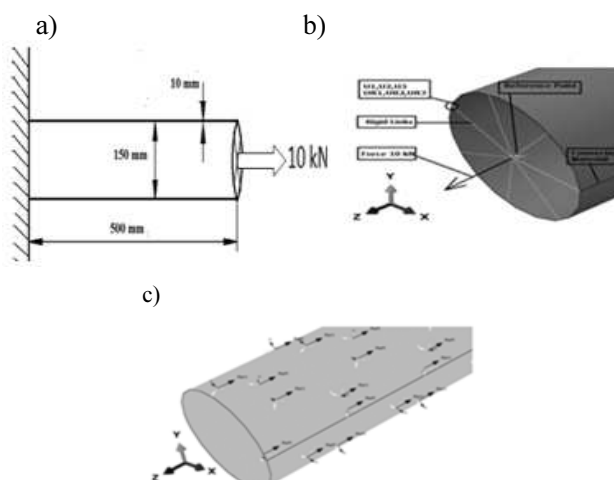


Fig. 4. Tubular element: a) diagram, b) load, c) element in ABAQUS (with and correction curve coordinate system)

Rys. 4. Element rurowy: a) schemat, b) obciążenie, c) element w programie ABAQUS (z założonymi i naniesionym krzywoliniowym układem współrzędnych)

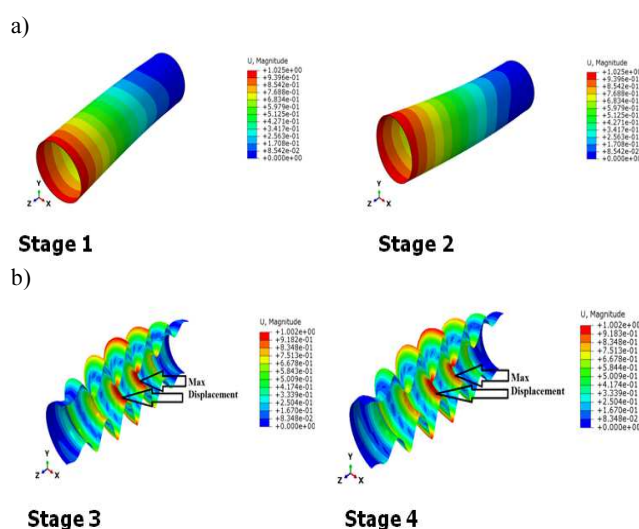


Fig. 5. Steps of modeling of deformation in direction of $-y$ and $+y$ axes (a) and slow twisting (b) until break (as illustrated by cross-sectional area)

Rys. 5. Etapy modelowania odkształceń w kierunku osi $-y$, i $+y$ (a) i powolnym skręcaniu (b) aż do jego zerwania (co ilustrują przekroje rury)

RESULTS ANALYSIS

With the assumption that in the model the load is uniform, in the first stage of the simulation cracking of the matrix occurs (Fig. 6a), which is characterized by an increased level of stress (line- \blacksquare - Fig. 6a), and then breakage of the reinforcement (stage 2 - the displacement increases considerably to approx. 1 mm). The location of the torn apart components (fibres) is quite difficult to verify since their distribution is incidental, and the process of destruction increases due to increased load among the reinforcement (fibres). This is more pronounced when twisting (Fig. 6b), causing a reduction in the level of stress in the laminate, and then an avalanche of destruction of the components, especially the reinforcement. The destruction process continues (i.e. increasing the load in the laminate components - in the fibres by the value of the rearranged part of the stress), causing the reinforcement to crack until the laminate strength limit is exceeded (line \blacktriangleleft Fig. 6a).

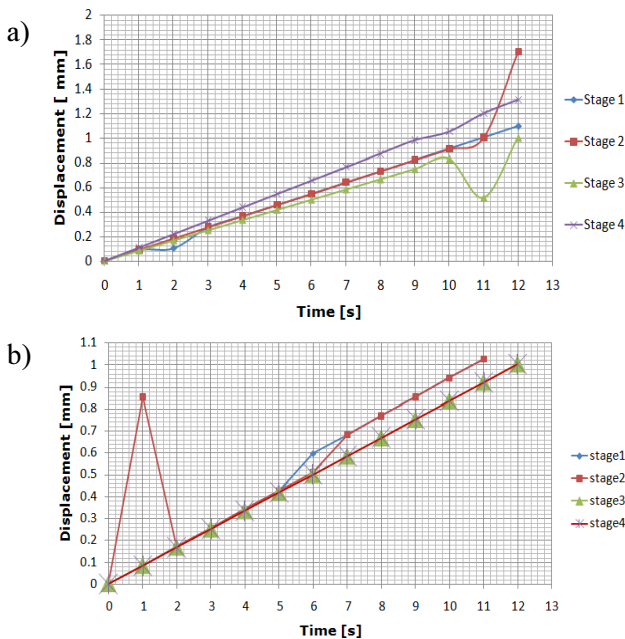


Fig. 6. Dependence of deformation stress levels as a function of time in tubular member (a) subjected to torsion (b)

Rys. 6. Zależność poziomów naprężeń od odkształcenia w funkcji czasu w elemencie cylindrycznym (a) poddanym skręcaniu (b)

In the modeled example, a further reduction in displacement will follow in the destruction process (2 stage) as a result of twisting. Stages 3 and 4 show the disappearance of resistance of the components (matrix and fibres, i.e. respectively, line \blacktriangleleft i \blacktriangleleft Fig. 6a), and hence a significant increase to the value of maximum displacement.

The process of destruction progresses in such conditions gradually preceding the perforation of emerging local discontinuities, whose accumulation leads to delamination, loss of tightness, and finally breaking of the fibers and the appearance of cracks.

CONCLUSIONS

Tubular model loads obtained using concentrated force applied centrally to Rigid Links allow one to obtain very accurate results at no additional cost. In addition, the change in the degrees of freedom in Rigid Links approximates the behavior of the structure to the way it works in natural conditions. Analyzing similar results on traditional universal testing machines was very difficult to achieve due to the implementation of a single-axle stretching test on them, which does not take into account changes in the degrees of freedom. In practice, however, thin-walled composite elements are exposed to work in a multi-axis stress state, which results in the formation of the above-mentioned stress and sometimes even deformation (Fig. 7).

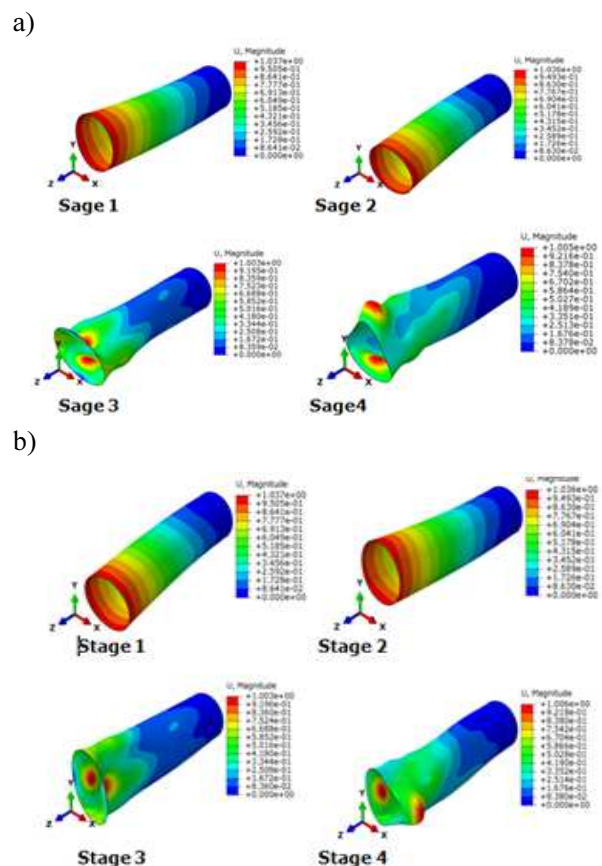


Fig. 7. Stages of deformation modeling at release of degrees of freedom in direction x (a) and y (b)

Rys. 7. Etapy modelowania odkształceń przy zwolnieniu stopni swobody w kierunku x (a) i y (b)

REFERENCES

- [1] Leda H., Kompozyty polimerowe z włóknami ciągłymi, Wyd. PP, Poznań 2000.
- [2] Savova L.N., Two-dimensional problem of the moment theory of viscoelasticity concerning stress concentration near circular hole, *Prikladnaja Mekhanika* 2010, 4, 6-13.
- [3] Żuchowski R., Wytrzymałość materiałów, Oficyna Wyd. PW, Wrocław 1996.
- [4] Ochelski S., Metody doświadczalne mechaniki kompozytów konstrukcyjnych, WNT, Warszawa 2004.

- [5] Chatys R., Mechanical properties of polymer composites produced by resin injection molding for applications under increased demands for quality and repeatability, *Journal of Ultrasound* 2009, 64(2), 35-38.
- [6] German J., *Podstawy mechaniki pękania*, Wyd. AGH, Kraków 2011.
- [7] Katunin A., *Degradacja cieplna materiałów polimerowych*, Wyd. ITE-PIB, Gliwice 2012.
- [8] Timoshenko S., *Teoria sprężystości*. Wyd. Arkady, Warszawa 1962.
- [9] Wierzbicki T., Bao Y., Lee Y.W., Bai Y., Calibration and evaluation of seven fracture models, *Inter. Journal of Mechanical Science* 2005, 47, 719-743.
- [10] www.budsoft.com.pl
- [11] Rusiński E., Czmochoński J., Smolnicki T., *Zaawansowana metoda elementów skończonych w konstrukcjach nośnych*, Oficyna Wyd. PW, Wrocław 2000.
- [12] Bodaszewski L., *Wytrzymałość materiałów z elementami konstrukcji*, Wyd. PŚk., Kielce 2005.
- [13] Кроссман Ф.В., Анализ разрушения слоистых композитов у свободного края. Разрушение композитных материалов, *Механика комп. матери* 1979, 2(25), 280-290.
- [14] Chatys R., Piernik K., Wyznaczenie właściwości mechanicznych kompozytów wzmocnionych włóknami, *Przetwórstwo Tworzyw* 2015, 6(149), 440-445.
- [15] Wierzbicki T., Xue L., On the effect of the third invariant of the stress deviator on ductile fracture. *Impact and Crashworthiness Lab Report #136*, 2005.
- [16] <http://help.autodesk.com/>.