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RESIDUAL STRESSES IN MULTILAYERED COMPOSITES - GENERAL OVERVIEW

The appearing process-induced stresses influence the mechanical performance of a composite structure and can initiate pre-load damage. Residual stresses evaluation is particularly important in multilayered composites where the ply orientations and stacking sequences highly influence the appearing stresses. Various numerical methods are used to simulate the growth and development of arising residual stresses. The general aim of the current work is to present the fundamental relations to predict the strains and stresses during the manufacture of laminated composites. Additionally, different modeling techniques and constitutive models are presented with the intention of understanding the different phenomena taking place during processing and which models best predict the process effects. As an example, a simple finite element model of a thermally loaded laminate is proposed to present the heterogeneities in through-thickness residual stresses distribution.

Keywords: residual stresses, multilayered composites, modeling, finite element method

NAPRĘŻENIA RESZTKOWE W KOMPOZYTACH WIELOWARSTWOWYCH - PRZEGLĄD ZAGADNIENIA

Obecność naprężeń poprocesowych wpływa na charakterystykę mechaniczną struktury kompozytowej i może inicjować wstępne zniszczenie. Oszacowanie wartości naprężeń resztkowych jest szczególnie istotne w kompozytach wielowarstwowych, w których konfiguracja ma znaczny wpływ na pojawiające się naprężenia. Istnieje wiele modeli opisujących powstawanie i rozwój naprężeń resztkowych. Celem prezentowanej pracy jest przedstawienie podstawowych związków stosowanych w opisie naprężeń powstałych w trakcie procesu wytwarzania. Zaprezentowano różne techniki modelowania oraz związki konstytutywne z zamiarem opisu różnorodnych zjawisk pojawiających się w trakcie wytwarzania laminatów. W celu wstępnej prezentacji heterogenicznego rozkładu naprężeń resztkowych przedstawiono model numeryczny wielowarstwowej płyty kompozytowej obciążonej termicznie.

Słowa kluczowe: naprężenia resztkowe, kompozyty wielowarstwowe, modelowanie, metoda elementów skończonych

INTRODUCTION

Currently, laminated composites are often used in modern construction, whose possible damage may have a catastrophic ending. Therefore, from the engineering point of view, it is important to predict the mechanical and non-mechanical stresses that influence damage initiation and evolution. In polymeric composites, process-induced stresses, so-called residual stresses, appear because of the thermal expansion mismatch between the matrix and fiber during cooling, chemical shrinkage of the resin during polymerization, non-uniform curing of the laminate [1-9], post-moisture absorption [10] in addition to mechanical tool-part interactions [11]. Residual stresses cannot be neglected because they may be high enough to initiate delaminations and micro-cracking inside the matrix even before loading [2, 7] or laminate warpage [11]. The appearance of residual stresses may drastically change the further fatigue be-

havior of constructions. For instance, residual stresses shift the First-Ply-Failure envelope, buckling loads, etc., and depending on the form of loading, residual stresses can increase or decrease fatigue life [12-17].

Regarding fibrous polymeric composites, residual stresses are present on three different levels [18], i.e., the microscale - the fiber-matrix scale, the ply scale and the laminate scale. Microscale residual stresses are caused by volumetric contraction of the matrix, discrepancy in thermal contraction between the constituents (matrix and fiber) after cool-down, and non-uniform consolidation or solidification [6]. Ply scale stresses arise because of the anisotropic behavior of longitudinal and transverse thermal expansion coefficients between the plies. The laminate (global) scale takes into account the variation of residual stresses through the thickness of the laminate due to the differ-

ence in shrinkage. There are two general sources of residual stresses: intrinsic (material, lay-up or structure shape) and extrinsic (processing and tooling). The intrinsic sources create residual stresses at the fiber-matrix level and consequently act from the inside to the outside. The extrinsic sources create stresses at the boundaries of the laminate structure and consequently act from the outside to the inside. Generally, intrinsic stresses have the largest influence on the strength, whereas extrinsic stresses control the shape of a composite structure [19].

Different techniques describe residual stresses such as experimental methods, analytical and numerical approaches. The experimental measurements involving destructive and nondestructive methods [20-29] are not discussed in the current paper. Several theoretical approaches have been applied in order to determine the residual stresses in polymeric composites including macrolevel or meso/microlevel models [1-5, 30-35]. Considering the macrolevel, laminate theory is applied in order to assess the processing stresses. Many different techniques exist for determining composite meso/microlevel behavior, i.e., the rule of mixtures, self-consistent mechanics approaches, the numerical method using representative volume elements (RVE) or the boundary element method [35]. The numerical predictions of internal stresses appearing during composite manufacturing help to investigate the effects of processing parameters on the mechanical integrity of a final part. Accurate description of the materials is vital for effective numerical analysis of phenomena which determine the generation of process-induced stresses and strains [36].

The most common, yet uncomplicated residual stress analyses assume that the composite is stress free at the cure temperature and then residual stresses occur during cooling from the cure temperature to room temperature [37]. Based on the above assumption, only the thermal effects are studied and chemical shrinkage is neglected. This is, however, not true for cases where the polymer is subjected to a three-dimensional stress state, such as laminated composites of a complex shape [38]. In that case, significant stresses may build up due to chemical shrinkage during cure. Considering residual stresses growth during the whole cure cycle, more accurate analysis is possible [3, 34, 35, 37, 38].

In order to obtain appropriate results, the mechanical constitutive model used in process modeling should be able to capture the mechanisms suitable for residual stresses formation. The mechanical constitutive models that have been used for residual stresses calculation consist of both elasticity and viscoelasticity. The incremental elastic relations have been presented, e.g. in paper [39]. The most accurate models for thermosets during processing based on the viscoelastic relations have been used, e.g. in [40-42], however, the viscoelastic behavior during curing is thermorheologically complex and it is difficult to model accurately for a layered composite.

Currently, the influence of nanostructure additions such as multi-walled carbon nanotubes [43] or nanofibers [44] on the residual stresses in fibrous composites is being studied. The results indicate that the addition of small amounts of nanostructures leads to a significant lowering of thermal residual stress on both micro and macrolevels.

The objective of this work is to present the different modeling techniques and fundamental relations used to assess residual stresses and strains in laminated composites. Given that only the thermo-mechanical aspects of the process will be considered in the current analysis, flow related phenomena will not be discussed. Generally, it is assumed that in cure modeling the laminate is perfectly filled and that no voids exist. Additionally, as an example of an anisotropic distribution of thermal residual stresses in layered composites, the simple thermally loaded elastic model of a plate will be presented. This numerical analysis is a preliminary work considering the residual stresses in the more complex analysis outlined in the last section of the current paper.

RESIDUAL STRESSES MODELING

RTM processes are used in composite part thermo-setting processing. The resin is injected into a heated mold with a fiber preform and cured under a controlled temperature. During the thermo-chemical process of curing, residual stresses arise because of both the spatial and temporal gradients of temperature and the degree of cure. The prediction of residual stresses in the curing process is coupled thermal-stress analysis because the stress solution is conditioned by the temperature field and degree of cure.

Residual strains

During and/or after the manufacturing process, the development of process-induced strains and the mismatch of these strains between the different components lead to the formation of residual stresses. In general, process-induced strains $\tilde{\varepsilon}_j^{free}$ are divided into three categories [45]: strains due to thermal changes (i.e., thermal expansion/shrinkage), strains due to phase changes in the matrix (i.e., cross-linking or crystallization) and strains due to moisture absorption, i.e:

$$\tilde{\varepsilon}_j^{free}(t) = \gamma_j(\alpha, T, M) \Delta T(t) + \beta_j(\alpha, T, M) \Delta M(t) + \phi_j(\alpha, T, M) \Delta \alpha(t) \quad (1)$$

where: j - the strain component, γ_j - the thermal expansion coefficient, β_j - the moisture expansion coefficient, ϕ_j - the cure expansion coefficient, M - moisture, T - temperature, α - degree of cure, Δ - increment in real physical time t . In the further part of the work, the influence of stresses/strains due to moisture absorption will be neglected.

Thermo-chemical model

Before assessing the process-induced stresses and strains, the knowledge of heat transfer is necessary. Based on the time dependent energy balance and the Fourier heat transference law, the thermo-chemical model is given as follows [46]:

$$\rho_c c_p \frac{\partial T}{\partial t} = k_{xx} \frac{\partial^2 T}{\partial x^2} + k_{yy} \frac{\partial^2 T}{\partial y^2} + k_{zz} \frac{\partial^2 T}{\partial z^2} + \dot{q} \quad (2)$$

where T and t are the temperature and time, x, y, z represent the spatial coordinates in the Cartesian system. ρ_c, c_p, k_{ij} are the density, the specific heat and the anisotropic thermal conductivity of the composites, respectively, and \dot{q} is the rate of heat generation from the chemical reaction.

The rate of cure reaction is assumed as proportional to the rate of heat generation and can be expressed as [46]:

$$\frac{d\alpha}{dt} = \frac{1}{\rho H_u} \dot{q} \quad (3)$$

where ρ is the polymer density, H_u is the total heat generated during the chemical reactions. Kinetic models are applied to describe the cure behavior of a polymer generally relating the cure rate to functions of temperature and cure degree as [47]:

$$\frac{d\alpha}{dt} = k(T) f(\alpha) \quad (4)$$

where $k(T)$ is the temperature dependent rate constant and $f(\alpha)$ is the conversion function, which describes the shape of the heat flow curve. The temperature dependence on the reaction rate is generally defined through an Arrhenius equation:

$$k(T) = A \exp(-E / RT) \quad (5)$$

where A is the frequency factor, E is the activation energies, R is the universal gas constant and T is the absolute temperature. A and E can be obtained by DSC [46]. The form of conversion function $f(\alpha)$ depends on the thermosetting polymer. In general, the cure behavior of thermosetting materials is usually described by n -th order or autocatalytic models [46, 48, 49]. Having relations for the thermo-chemical behavior of the composite during the curing process, other properties including temperature and cure degree can be determined readily in an incremental way, well suited for numerical implementation.

Incremental finite element approach

The numerical formulation is based on the integral expression of Eq. (2) written as [46]:

$$\int_V (\rho_c c_p \frac{\partial T}{\partial t}) \delta T dV = \int_V (k_{xx} \frac{\partial^2 T}{\partial x^2} + k_{yy} \frac{\partial^2 T}{\partial y^2} + k_{zz} \frac{\partial^2 T}{\partial z^2} + \dot{q}) \delta T dV \quad (6)$$

In Eq. (6) the temperature and degree of cure at an arbitrary position in an element can be formulated as node temperature and the degree of cure:

$$T = \sum_{i=1}^m N_i T_i, \quad \alpha = \sum_{i=1}^m N_i \alpha_i \quad (7)$$

where T_i and α_i are the node temperature and the node degree of cure, respectively, and N_i is the element form function. The temperature and degree of cure vary with time and position and induce internal stresses. Therefore, the mechanical properties of the composite depend on the temperature and the degree of cure, e.g. [5, 50]. The degree of cure solved by the pure heat transfer problem will be next added to the residual stress analysis.

Viscoelastic model

Using the classical linear viscoelastic approach, e.g. [40], the stress components can be evaluated from the following relation:

$$\sigma_i(t) = \int_{-\infty}^t Q_{ij}(t, \tau) \frac{\partial}{\partial \tau} [\varepsilon_j(\tau) - \tilde{\varepsilon}_j^{free}(\tau)] d\tau, \quad i, j = 1, 2, \dots, 6 \quad (8)$$

where Q_{ij} denotes the matrix stiffness components. For the given value of cure parameter α_c [45], relation (8) can be rewritten in the form:

$$\sigma_i(t) = \int_{-\infty}^t Q_{ij}(\alpha_c, \xi - \xi^i) \frac{\partial}{\partial \tau} [\varepsilon_j(\tau) - \tilde{\varepsilon}_j^{free}(\tau)] d\tau, \quad i, j = 1, 2, \dots, 6 \quad (9)$$

$$\text{where: } \xi = \int_0^t \chi[\alpha_c, T(s)] ds, \quad \xi^i = \int_0^{\tau} \chi[\alpha_c, T(s)] ds$$

The mechanical properties of the composite material change as curing progresses. In particular, the transverse compliance, $S_{22}(t) = [Q_{22}(t)]^{-1}$, undergoes a substantial change with time during curing [50]. This behavior can be modeled by a power law in the form:

$$S_{22}(\alpha, t) = S_{22i}(\alpha) f(t) + D(\alpha) [\chi_T(\alpha, T)]^{q(\alpha)} \quad (10)$$

where $D(\alpha) = D_i + (D_f - D_i)\alpha$, $q(\alpha) = q_i + (q_f - q_i)\alpha$, $f(t)$ is a material dependent function chosen to agree with the results [3], D the transverse creep coefficient, αT the shift factor and q is the transverse creep exponent. Assuming the Young's modulus varies with cure parameter α and the form of the cure parameter varies with time, one can obtain resultant stresses for the given

laminate stacking sequences and their variations with time. The correctness of these models depends on how well the material property variables fit the experimental data - generally increasing the number of discrete fitting variables increases accuracy.

Plane stress model

Using the classical plane stress relations for elastic thin laminates for which all out-of-plane normal and shear stress components become negligible, we can calculate the stresses as follows (see also Eq. (1)):

$$\sigma_i = Q_{ij}(\varepsilon_j - \tilde{\varepsilon}_j^{free}), \quad i, j = 1, 2, 6 \quad (11)$$

Strains are assumed to change linearly through the thickness. Having information about the global lamina process induced strains and stresses, from laminate theory the normal loads and bending moments are found as:

$$N_i = \int_{-t/2}^{t/2} \sigma_i dz, \quad M_i = \int_{-t/2}^{t/2} \sigma_i z dz, \quad i = 1, 2, 6 \quad (12)$$

and writing relations (11) and (12) in the incremental form we finally obtain the midplane strain increment values:

$$\sum_{k=1}^{NL} Q_{ij}^k t_k d\varepsilon_j^0 - \left(\sum_{k=1}^{NL} Q_{ij}^k t_k d\tilde{\varepsilon}_j^{0, free} \right) = 0 \quad (13)$$

the positions of neutral axis z_b :

$$\sum_{k=1}^{NL} Q_{ij}^k (t_k - z_b) \delta_{jr} = 0 \quad (14)$$

and the curvature increments:

$$\sum_{k=1}^{NL} \int_{z_{k-1}}^{z_k} dz Q_{ij}^k [d\varepsilon_j^0 + (z - z_b) d\kappa_j - d\tilde{\varepsilon}_j^{0, free}] = 0 \quad (15)$$

where ε_j^0 is the vector of midplane strains and κ_j is the vector of curvature components. The above relations were derived under the assumption that the mould is not subjected to any constraints.

EFFECTIVE PROPERTIES OF COMPOSITE

The effective mechanical and thermal properties are determined from the fiber and matrix properties, the fiber volume fraction and orientation. Aboudi et al. [51, 52] presented an extension of the self-consistent field method (SCF). It relies on the fundamental assumption that a two-phased composite has a periodic structure in which the reinforcing material (e.g., fibers)

is arranged in a periodic manner, thus forming a periodic array (Fig. 1).

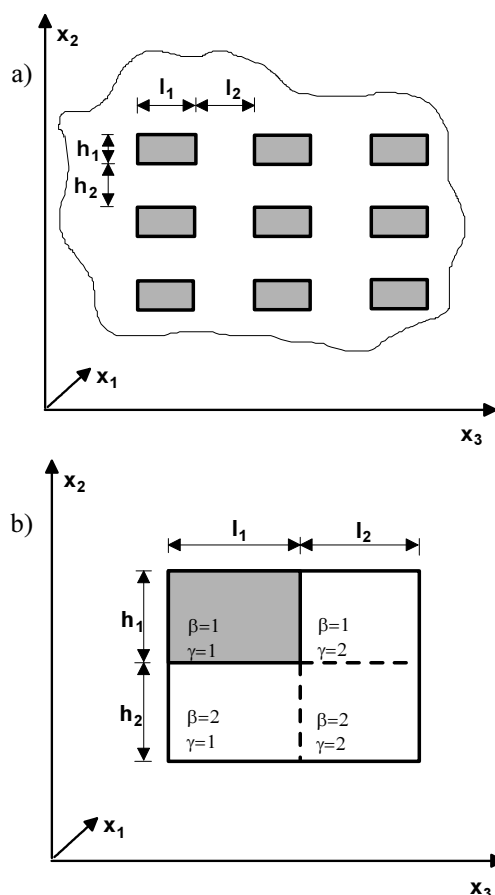


Fig. 1. Model of continuously reinforced fiber composite: a) a composite with doubly periodic array of fibers extending in the x_1 -direction, b) RVE with four subcells

Rys.1. Model kompozytu wzmocnionego włóknami ciągłymi: a) kompozyt z podwójnie periodycznym obszarem włókien wydłużonych w kierunku x_1 , b) komórka reprezentatywna z czterema podobszarami

The fiber cross-section is denoted by $h_1 l_1$, whereas h_2 and l_2 are the distances between the adjacent fibres. The relations for the average stresses and strains are presented as follows:

$$\langle \varepsilon_{ij} \rangle = \frac{1}{V_1} \sum_{\beta, \gamma=1}^2 v_{\beta\gamma} \varepsilon_{ij}^{(\beta\gamma)} \quad (16)$$

$$\langle \sigma_{ij} \rangle = \frac{1}{V_1} \sum_{\beta, \gamma=1}^2 v_{\beta\gamma} N_{ij}^{(\beta\gamma)} \quad (17)$$

where $v_{\beta\gamma} = h_\beta l_\gamma$ is the area occupied by the subcells, and $V_1 = (h_1 + h_2)(l_1 + l_2)$ is the total area of RVE, $N_{ij}^{(\beta\gamma)}$ are the stresses in the subcells computed as follows:

$$N_{ij}^{(\beta\gamma)} = \frac{1}{V_1} \int_{-h_\beta/2}^{h_\beta/2} \int_{-l_\gamma/2}^{l_\gamma/2} \sigma_{ij}^{(\beta\gamma)} dx_2^{(\beta)} dx_3^{(\gamma)} \quad (18)$$

Based on the above relations, the effective material constants for a transversely isotropic body can be computed. Additionally, for geometric relation $l_1/l_2 \rightarrow \infty$, the lamina configuration is present with two periodic subcells having h_1 and h_2 thickness.

NUMERICAL EXAMPLE

In order to present the anisotropic thermal residual stresses distribution through the thickness of the multilayered composite, the finite element model has been prepared. An 8-ply composite plate having all plies of a constant thickness made of unidirectional CFRP with an 0.55 fibre volume fraction was analyzed [53]. To simulate confinement in a mould, fixed boundary conditions were applied. In order to present a through-thickness residual stresses profile versus ply orientations and stacking sequence, four lamina configurations were selected: [0/90/0/90]_s, [0/45/-45/90]_s, [0/30/-30/90]_s, [0/75/-75/90]_s. The plate was subjected to a thermal load simulating the cool-down process from processing temperature to room temperature assuming a thermal drop equal to -100°C .

Only example results are presented here. The variation of residual stresses σ_1 through the plate thickness (Fig. 2) revealed a considerable influence of the ply orientations and the stacking sequence. The through-thickness anisotropy in the residual stress profile is visible. The observed thermal residual stresses were in the range from more than 13 MPa up to about 25 MPa depending on the composite configuration.

The present simple finite element model considering only the last step of thermoforming (cool-down) is used in the start analyses of residual stresses, which can help

to eliminate the most unfavorable lamina configurations. It can also be applied as to quickly assess the accuracy of process-induced stresses values obtained experimentally.

PLANNED FUTURE RESEARCH

The present numerical analysis is the beginning step in a more detailed analysis of process-induced stresses modeling. With the intention of understanding the different phenomena taking place during processing, numerical analysis of process-induced stresses of laminates will be performed using the finite element method. Starting from solving the pure heat transfer problem of the curing process, the obtained temperature and cure degree distributions will be incorporated into a stress analysis as a predefined field. During the residual stress analysis, the mechanical properties of the materials will be defined as a function of the degree of cure.

Next, we intend to study in detail the fatigue process for laminated composite structures that incorporates the initial pre-stressing of a composite and then to add fatigue loading and evaluate the failure loads using, e.g. the Tsai-Wu criterion. The effective properties of a multilayered composite will be evaluated based on a representative volume element (RVE) with specific boundary conditions proposed in the work of Chwał and Muc [54]. For the arbitrary plies orientations of a lamina, the residual stress damage curves can be evaluated. Since it is possible to control the temperature and time variations during curing, we can describe how the temperature influences the failure envelope (the translations of the failure envelope) (Fig. 3).

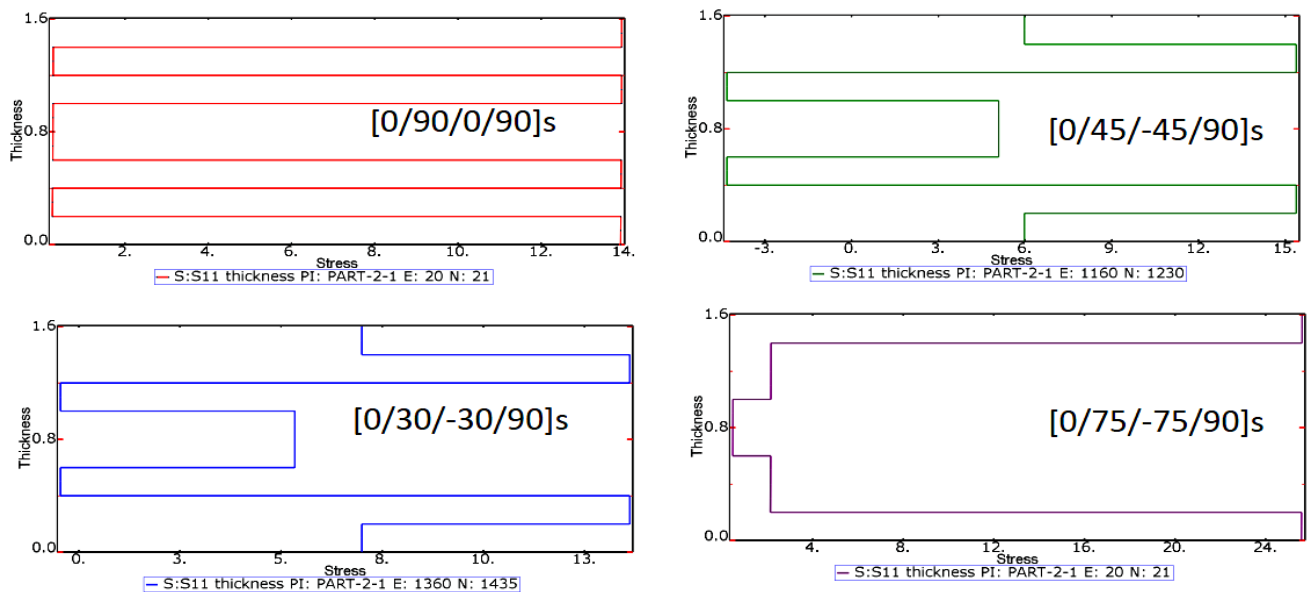


Fig. 2. Through-thickness residual stress profiles σ_1 [MPa] of laminate for different configurations
 Rys. 2. Rozkład naprężeń resztkowych σ_1 [MPa] po grubości laminatu dla różnych konfiguracji

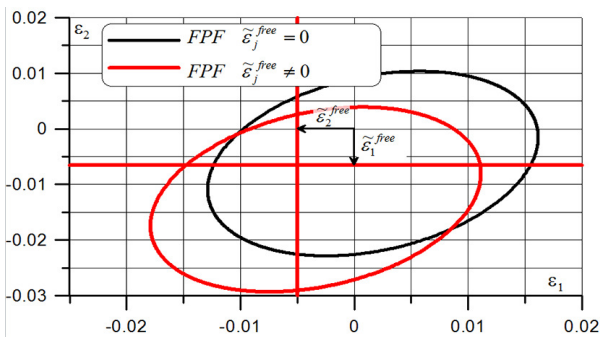


Fig 3. FPF envelope shift caused by residual strain (Tsai-Wu) [45]

Rys. 3. Przesunięcie powierzchni zniszczenia spowodowane odkształceniami resztkowymi (Tsai-Wu) [45]

Based on the proposed approach, if the failure envelope is known, for a laminate subjected to cyclic loading, we can describe the changes in Young's modulus. Finally, we can evaluate the influence of the residual stresses on the altering of the Young's modulus and fatigue damage (see [55]). For composite structures subjected to fatigue loading, we intend to formulate and solve the optimization problem of the thermoforming process by controlling the process conditions being design variables.

CONCLUDING REMARKS

The short introduction to the problem of residual stresses in layered composites has been presented, focusing mainly on the fundamental relations for predicting process-induced strains and stresses. Using the proposed methodology, accurate description of the generation of process-induced stresses is possible. Detailed studies of the numerical modeling of residual stresses in layered composites will be presented in the next paper.

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