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COMPOSITES THEORY AND PRACTICE

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STATIC AND FATIGUE STRENGTH OF COMPOSITE PLATES WITH HOLES

The number of structures made from multi-layered composite materials increases every year. Some of these structures include different cutouts, which result from the manufacturing process or are needed for maintenance. This kind of discontinuities are the source of stress concentrations, which cause the nucleation and evolution of various forms of damages in composite materials. The problem of the damage becomes particularly important when the structure is subjected to cyclic loads. The present work concentrates on the investigation of the strength evaluation for rectangular composite plates with internal holes of different shapes. Numerical results are presented for circular, elliptic and square (with rounded corners) holes placed in the centre of a plate, which is subjected to bi-directional tension pressure imposed along the outer edges. It is assumed that the stacking sequence is symmetric with respect to the middle surface of the structure. The considered plates are made from an angle-ply laminate, which consists of the twelve layers with a fiber orientation angle ±θ, for example $[\pm5^\circ, \pm5^\circ, \pm5^\circ, \pm5^\circ]_S$, where 's' denotes the symmetry. The fiber orientation angle θ is studied in order to find the maximum strength of the considered plate. The solution is sought from the following discrete values, namely [0°, 5°, 10°, 15°, … , 90°]. The numerical calculations are performed with the use of the multipurpose finite element code ANSYS 12.1. In order to estimate the static strength of the structure, the linear (admissible stress) first ply failure criteria are applied. The influence of the geometry of the plate and the cutout on the optimal solution (fiber orientation angle ±θ) is also investigated. It is assumed that the ratio of the area of the analyzed plate and the hole is constant for the different shapes of holes. Simulations are performed for the two different materials, where the ratio of Young's modulus E_1, E_2 equal, respectively, $E_1/E_2 \ll 1$ (anisotropy) and $E_1/E_2 \approx 1$ (quasi-isotropy).

Keywords: laminate, failure criteria, finite element method, stacking sequence optimization, damage, plate with hole

STATYCZNA I ZMĘCZENIOWA WYTRZYMAŁOŚĆ KOMPOZYTOWYCH PŁYT Z OTWORAMI

Obecnie z wielowarstwowych materiałów kompozytowych wykonywanych jest coraz więcej elementów konstrukcji. Wiele z tych elementów posiada rozmaite otwory niezbędne w procesie montażu, a następnie w procesie eksploatacji. Tego typu nieciągłości stanowią główną przyczynę powstawania koncentracji naprężeń. Zjawisko to z kolei jest przyczyną powstawania i narastania uszkodzeń, szczególnie przy obciążeniach zmiennych w czasie. Z tego też powodu niezwykle istotne staje się zagadnienie optymalnego projektowania tychże materiałów z punktu widzenia wytrzymałości zmęczeniowej. W pracy rozważano płaski, prostokątny element kompozytowy z centralnie zlokalizowanym kołowym, eliptycznym lub kwadratowym otworem. Element ten poddany jest działaniu ciśnienia przyłożonego na krawędziach zewnętrznych. Obciążenie to ma charakter proporcjonalny i zawarte jest w płaszczyźnie elementu. Założono symetryczną względem powierzchni środkowej konfigurację laminatu typu *angle - ply*, złożoną z ośmiu warstw wykonanych z identycznego materiału. Celem optymalizacji jest wskazanie takiej wartości kąta orientacji włókien ±θ, która gwarantuje największą możliwą wytrzymałość konstrukcji statyczną konstrukcji, a co za tym idzie, również zmęczeniową. Rozwiązanie poszukiwane jest w zbiorze wartości dyskretnych [0°, 5°, 10°, 15°, … , 90°]. Do wykonania niezbędnych obliczeń wytrzymałościowych wykorzystano komercyjny system oparty na metodzie elementów skończonych ANSYS 12.1. Do oszacowania nośności konstrukcji wykorzystano liniowe (naprężeniowe) kryterium zniszczenia typy *first ply failure*. Analizie poddano również wpływ zarówno wymiarów geometrycznych płyty, jak i otworu na optymalne wartości kąta orientacji włókien, przy czym założono, że pole powierzchni rozważanych płyt oraz otworów jest stałe. Symulacje przeprowadzono dla dwóch materiałów, dla których stosunek wartości modułów Younga wynosi odpowiednio $E_1/E_2 \ll 1$ oraz $E_1/E_2 \approx 1$.

Słowa kluczowe: laminat, kryteria zniszczenia, metoda elementów skończonych, uszkodzenie, optymalizacja, płyta z otworem

INTRODUCTION

Various parts of support structures contain different types of cutouts. The main disadvantage connected with the presence of holes is the stress concentrations in the vicinity of the edges of the holes. This phenomenon causes a reduction in the static strength of the structure. In case of cyclic load, when the applied loads are much lower than the static strength of the structure, the stress concentration leads to a rapid accumulation of damage in the composite material and results in premature failure of the whole structure. Reduction of the stress concentration magnitude can be achieved by changing the shape of the cutout [1, 2], or by the application of additional reinforcing elements [3, 4], or by the optimal selection of fiber orientation angles in the composite material. It seems that the last proposal is the most attractive from the technological point of view, particularly in the case of the angle-ply configuration of the fibers in the laminate. On other hand, considering the rapid development of commercially available software (based on the finite element method) static analysis of the composite structure becomes increasingly easier. Moreover, now it is possible to very quickly analyze a wide range of structures in order to select the optimal one. The present work can be regarded as an introduction for further studies on the evaluation of the fatigue life of real composite structures, particularly from the point of view of applying the ANSYS system in the case of modeling and simulating composite structures.

STRUCTURE

The investigated structure is a rectangular composite plate with dimensions a x b and constant thickness t_0. In the geometrical center of the structure a cutout is introduced. The convenient way to describe the boundary of the applied cutouts is proposed in the parametric form in [5], as follows:

$$\begin{aligned} x(\varphi) &= \lambda_0 \left[\cos(\varphi) + w_0 \cos(n_0 \varphi)\right] \\ y(\varphi) &= \lambda_0 \left[c_0 \sin(\varphi) - w_0 \sin(n_0 \varphi)\right] \end{aligned} \tag{1}$$

where λ_0 controls the size of the hole, n_0 and c_0 determines the shape and w_0 is the bluntness factor which changes the radius of the curvature at the corners of the cutout. The analyzed plate with different shapes of holes is presented in Figure 1. It is also assumed that the ratio of the area of the hole and the area of the plate, for different shapes of cutouts, is constant (A_h/A_p = const). Additionally, due to the symmetry of the structure, only 1/4 of the plate is taken under consideration for the computations.

The total thickness of the plates is equal to $t_c = 2.5$ mm. The composite material consists of 12 layers. It is assumed that the material of the each layer is identical. Two different types of material for the layers are considered, namely carbon fibers with epoxy resin and carbon fabric with an epoxy resin. The first of them is highly anisotropic ($E_2/E_1 = 0.057$) while the second one has "quasi" isotropic mechanical properties ($E_2/E_1 = 1$). The mechanical properties of these materials [6] are shown in Table 1. Furthermore, the strength coefficients (admissible stresses) are also presented in Table 2. The laminate has an angle - ply configuration ±θ. In order to avoid any bending effects connected with the properties of the internal structure, the stacking sequence is symmetric with respect to the middle surface of the plate, which means that the following plies are oriented as follows: $[-\theta, +\theta, -\theta, +\theta, -\theta, +\theta]_S$, where 's' denotes the symmetry.

TABLE 1. **Mechanical properties of individual layers**
TABELA 1. **Własności mechaniczne warstwy indywidualnej**

Material	E_1 [GPa]	E_2 [GPa]	G_{12} [GPa]	ν_{12}
Carbon fibers/ epoxy resin	181	10.3	7.17	0.28
Carbon fabric/ epoxy resin	74	74	4.55	0.15

E_1 - Young's modulus in fiber direction, E_2 - Young's modulus perpendicular to fiber, G_{12} - Kirchoff's modulus, ν_{12} - Poisson's coefficient.

TABLE 2. **Admissible stresses for individual layers**
TABELA 2. **Wartości naprężeń dopuszczalnych dla warstw indywidualnych**

Material	X_t [MPa]	X_c [MPa]	Y_t [MPa]	Y_c [MPa]	S [MPa]
Carbon fibers/ epoxy resin	1500	1500	40	246	68
Carbon fabric/ epoxy resin	499	352	458	352	46

X_t/X_c - tensile/compressive strength in fiber direction, Y_t/Y_c - tensile/ compressive strength perpendicular to fiber, S - shear strength.

The external edges are subjected to uniform tension in the horizontal and vertical directions. It is assumed that the proportional system of loading, namely pressure p_v in the vertical direction is equal to $p_v = k \cdot p_h$, where k is the constant parameter (proportional coefficient) and p_h denotes the load in the horizontal direction (see Fig. 1).

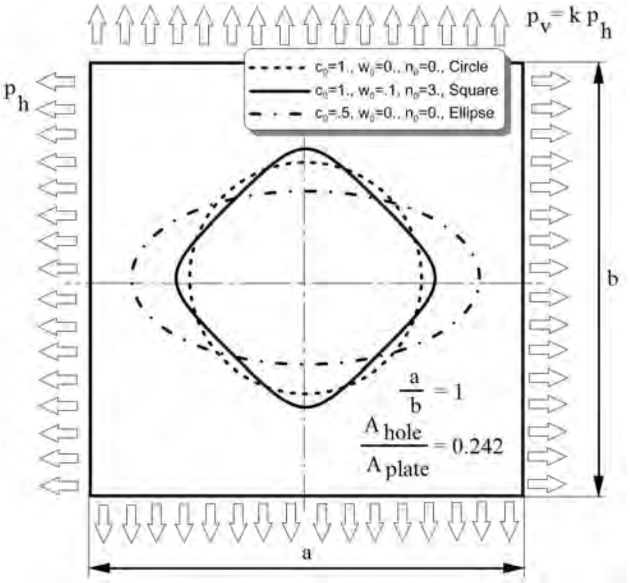

Fig. 1. Investigated composite plate with cutouts of different shape
Rys. 1. Kompozytowa płyta z otworami o różnym kształcie

FORMULATION OF OPTIMIZATION PROBLEM

The optimization problem can be stated as follows: find fiber orientation angle θ for which the strength of the investigated plate with a cutout is maximal. In other words, the plate with the optimal fiber orientation angle θ is able to carry the maximum load, namely:

$$p_h \to \max \tag{2}$$

It is assumed that the optimal value of angle θ is sought from the following discrete set [0°, 5°, 10°, 15°, … , 90°]. In order to determine the strength of the investigated structure, the linear (admissible stress) first ply failure criterion is applied. The admissible stresses for the individual layer materials are shown in Table 2.

From the practical point of view, it is convenient to formulate the optimization problem in the following manner. For the applied constant load, optimal fiber orientation angle θ leads to the minimum of the following function (criterion of admissible stresses):

$$\min_{\theta}\left[\max\left(\frac{\sigma_1(x,y)}{X_t},\ \frac{|\sigma_1(x,y)|}{X_c},\ \frac{\sigma_2(x,y)}{Y_t},\ \frac{|\sigma_2(x,y)|}{Y_c},\ \frac{|\tau_{12}(x,y)|}{S},\right)\right] \tag{3}$$

where σ_1, σ_2, τ_{12} are elements of the stress tensor evaluated in each point of the analyzed structure. The maximum in expression (3) is evaluated for the whole structure. In the case of the Tsai-Wu failure criterion, the form of expression (3) is quite different. However, this criterion is not currently used and therefore will not be discussed in detail now. Formulations (2) and (3) are equivalent. They lead to an identical solution, namely the same optimal value of angle θ. In the present work, approach (3) is used. In order to find the optimal solution, computations are performed for each feasible value of angle θ. The obtained results are then compared and the minimal value of expression (3) is found.

FINITE ELEMENT MODEL OF THE STRUCTURE

All the necessary computations are performed with use of a commercially available system based on the finite element method - ANSYS 12.1 Classic [7]. Nowadays, it is one of the most popular finite element packages. The Classic module contains several types of special shell or solid elements which are particularly dedicated to the modeling of multilayered composite structures.

The plate shown in Figure 1 is modeled with the use of composite shell elements SHELL181. They have six degrees of freedom in each node, namely transition in the X, Y, Z direction and rotation about the X, Y and Z axes. The solution is approximated by linear shape functions. Fiber orientation angle θ is measured with respect to the direction of the X axis of the local coordinate system connected with each finite element. These coordinate systems have to be appropriately defined before generating the finite element mesh. In Figure 2 the finite element mesh generated for the investigated plates with circular hole is depicted. The presented mesh consists of triangular shell elements only. The size of the element along edge AE is approximately two times smaller than in the rest of the structure. As has been mentioned, due to symmetry only 1/4 of the structure is taken under consideration for the computations and the appropriate symmetry conditions are applied. Along edges BC and CD the structure is simply supported. It means that the displacement in the Z direction is only constrained. Moreover, due to the symmetry of the structure, displacement UX along edge AB and UY along edge ED are also constrained. External pressure is applied on edges BC and CD.

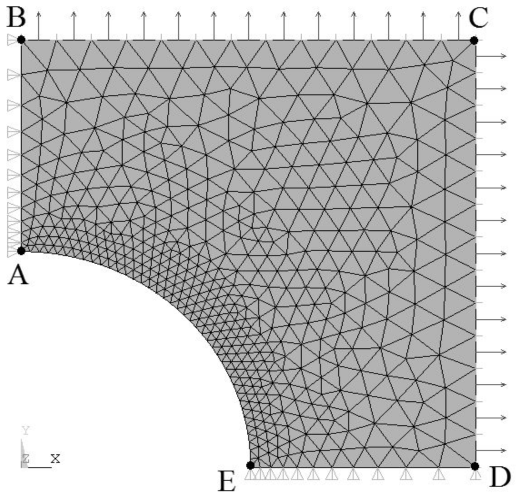

Fig. 2. Finite element model of composite plate

Rys. 2. Siatka elementów skończonych płyty

The convergence test of the finite element solution is performed on the plate with the square hole, as shown in Figure 1. The plate is made of carbon fiber/epoxy resin. The other parameters are $A_h/A_p = 0.2$, $b/a = 1$, $a = 198.166$ mm and the $\theta = 35$[°]. The applied pressure is equal to $p_v = p_h = 10.5$ MPa. In Table 3 the obtained results are presented. Starting from the model where the number of elements is equal to 1926, a satisfactory convergence of the numerical solution can be observed. For further computations, the model which contains exactly 1926 finite elements is used. In this model, the ratio of the length of the element edge and the length of the shorter edge of the plate is approximately equal to $l_e/l_p \approx 0.02$.

TABLE 3. **Results of convergence test of FE solution**

TABELA 3. **Wyniki testu zbieżności rozwiązania MES**

Criterion	**Number of elements (size of larger elements [mm])**				
	138 (16.0)	**490 (8.0)**	**1926 (4.0)**	**7651 (2.0)**	**30868 (1.0)**
Admissible Stress	0.259354	0.297121	0.312052	0.314802	0.315666
Tsai - Wu	0.275354	0.323763	0.354343	0.362061	0.362584

RESULTS OF OPTIMIZATION

The optimization is carried out for two cases. In the first case, the external dimensions of the analyzed composite plates are variable, namely b/a = 1.00, 0.778, 0.600, 0.454, 0.333. For all the analyzed structures, ratio $A_h/A_p = 0.2$. The applied tension load is also constant. In the second case, for the fixed external dimensions, $b/a = 1.0$, the ratio of the applied external pressure in the horizontal and vertical direction is variable, namely p_v/p_h = 1.0, 0.75, 0.50, 0.25, 0.00. The computations are repeated for the circular, elliptical and square hole shapes. The shapes of the holes are described by the parameters as shown in Figure 1. The obtained optimal values of fiber orientation angle θ are collected in Tables 4-7. The results presented in Tables 4 and 5 prepared in the case of the use of the anisotropic material (carbon fiber/epoxy resin), where $E_2/E_1 = 0.057$ and the results presented in Tables 6 and 7 are in the case of the use of the quasi isotropic material (carbon fabric/epoxy resin), $E_2/E_1 = 1.0$. The mechanical and physical properties of these materials are discussed above (see Tabs 1 and 2).

TABLE 4. **Values of optimal fiber orientation angle θ [°]**
TABELA 4. **Wartości optymalne kąta orientacji włókien θ [°]**

Shape of cutout	**Ratio b/a**				
	1.00000	**0.77778**	**0.60000**	**0.45455**	**0.33333**
Circle	45	45	45	40	35
Ellipse	60	60	55	50	40
Square	45	45	40	35	20

Material of layer carbon fiber/epoxy resin $E_1/E_2 = 0.057$, $A_h/A_p = 0.2$, $p_v/p_h = 1$, Criterion: Admissible Stress

TABLE 5. **Values of optimal fiber orientation angle θ [°]**
TABELA 5. **Wartości optymalne kąta orientacji włókien θ [°]**

Shape of cutout	**Ratio p_v/p_h**				
	0.00	**0.25**	**0.50**	**0.75**	**1.00**
Circle	25	30	30	35	45
Ellipse	30	30	45	55	60
Square	20	25	25	35	45

Material of layer carbon fiber/epoxy resin $E_1/E_2 = 0.057$, $A_h/A_p = 0.2$, $b/a = 1$, Criterion: Admissible Stress

TABLE 6. **Values of optimal fiber orientation angle θ [°]**
TABELA 6. **Wartości optymalne kąta orientacji włókien θ [°]**

Shape of cutout	**Ratio b/a**				
	1.00000	**0.77778**	**0.60000**	**0.45455**	**0.33333**
Circle	20	70	65	25	25
Ellipse	65	65	65	65	65
Square	25	65	65	15	0

Material of layer carbon fabric/epoxy resin $E_1/E_2 = 1$, $A_h/A_p = 0.2$, $p_v/p_h = 1$, Criterion: Admissible Stress

TABLE 7. **Values of optimal fiber orientation angle θ [°]**
TABELA 7. **Wartości optymalne kąta orientacji włókien θ [°]**

Shape of cutout	**Ratio p_v/p_h**				
	0.00	**0.25**	**0.50**	**0.75**	**1.00**
Circle	20	20	20	25	20
Ellipse	65	20	70	65	65
Square	25	25	25	25	25

Material of layer carbon fabric/epoxy resin $E_1/E_2 = 1$, $A_h/A_p = 0.2$, $b/a = 1$, Criterion: Admissible Stress

For the anisotropic materials (Tabs 4 and 5), the optimal values of fiber orientation angle θ significantly depend on ratio b/a as well as ratio p_v/p_h. The shape of the cutout also has influence on the optimal solution. Generally, together with a decrease in ratio b/a as well as p_v/p_h, the obtained optimal values of angle θ also become smaller.

The results obtained in the case of the use of the quasi isotropic material for the layers have quite different character. It is very difficult to observe any regularity. Generally, in the case of the variable value of ratio b/a, the optimal solutions are equal to approximately two values, namely 20, 25 and 65, 70 degrees. It is most likely caused by the very small difference between the strength of the material on tension in the X and Y direction (Tab. 2). However, in the case of the variable of ratio p_v/p_h, the optimal value of angle θ does not depend on ratio p_v/p_h. The optimum depends only on the shape of the hole.

In Figure 3, the relationship between fiber orientation angle θ and the maximal value of the (admissible stresses) failure criterion is presented. This graph was prepared for the plate with the square cutout, which is made of the anisotropic and quasi isotropic material.

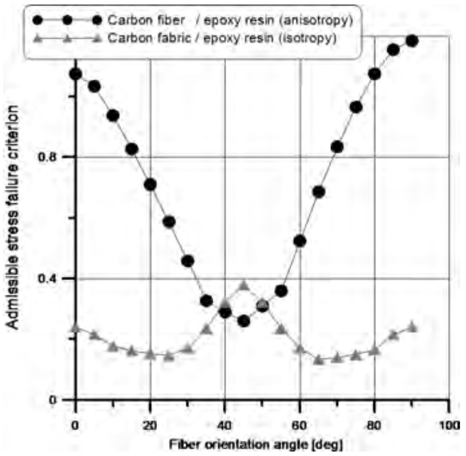

Fig. 3. Relationship between θ and maximal value of admissible stress failure criterion

Rys. 3. Zależność pomiędzy kątem orientacji włókien a maksymalną wartością naprężeniowego kryterium zniszczenia

The rest of geometrical and load parameters are the same as in case of the convergence test. It is worth stressing that in the case of the anisotropic material, the profit obtained from the optimization is very high. Moreover, in this case a single global minimum can be observed. In the case of the quasi isotropic material, the profit from the optimization is much lower. What is characteristic, two local minimums exist. This fact can explain the behavior of the optimal values of angle θ in Tables 7 and 8, which are obtained for the quasi isotropic material of the layers.

FATIGUE STRENGTH AND FATIGUE LIFE OF STRUCTURE

In the case when the composite structure is subjected to cyclic load, degradation of the mechanical properties is observed. The degradation of the mechanical properties, namely E_1, E_2 and G_{12}, generally depends on the number of load cycles. In Figure 4, there is a typical relationship between Young's modulus and the number of cycles, where nf denotes the number of cycles which cause the damage of the composite material, and E is the initial (virgin) Young's modulus.

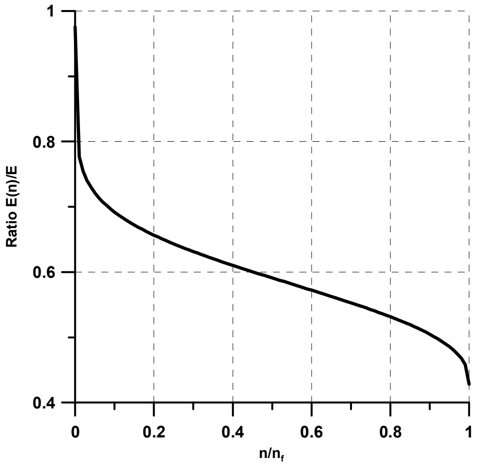

Fig. 4. Degradation of Young's modulus as function of number of cycles

Rys. 4. Degradacja wartości modułu Younga w funkcji liczby cykli

In the first phase, the process of stiffness reduction is very rapid. Afterwards, the evolution of this process is much slower. This phase is relatively long in comparison with the other two phases. At the end of the fatigue life of the structure, this process significantly accelerates, which leads to the total failure of the structure. This behavior is caused by the gradual growth of damage on the microscale, namely matrix cracking, fiber breaking, fiber debonding and delamination. This process also causes a reduction in the strength of the composite material and in consequence the strength of the whole structure.

In the proposed research, the spatial non-uniformity of the material properties at the microscopic level is to be taken into account from experimental data obtained during fatigue tests conducted for plies oriented at 0°, 45° and 90° in tension and compression. When processed, this information will be represented by the lower and upper boundaries of the stiffness degradation, i.e., as stiffness $E(N)$ versus the number of cycles N relationships. These sets of lower and upper limits will be available independently for 0°, 45° and 90° orientations. Next, three different scalar damage parameters, of global nature, are to be introduced with values between zero (virgin material state) and unity (final mode of failure). The damage parameters characterize and simulate the three stages of stiffness degradation (sharp initial decline-gradual deterioration-final failure) [8-10]. The above approach will facilitate damage (fracture) analyses to be conducted for individual plies in any arbitrarily laid-up laminate. To put it differently, this novel approach will allow one to utilize a mesomodel whereby existing FE modelling can be used.

CONCLUSIONS

In the case of the anisotropic properties of the layer material, the strength of the structure, which is estimated with the use of the admissible stresses failure criterion, is very sensitive to the direction of the fiber. The profit from the optimization is significant. In the case of the optimal structure, the maximal value of the failure criterion is several times smaller in comparison with the worst structure. In the case of the quasi isotropic layer material, profit from the optimization is also present but is not as significant as in the case of the anisotropic material.

REFERENCES

[1] Muc A., Effectiveness of optimal design with respect to computational models for laminated composite structures weakened by holes, Struct. Optim. 2000, 19,169-182.

[2] Pedersen P., Tobiesen I., Jensen S.H., Shapes of orthotropic plates for minimum energy concentration, Mech. Struct. 1998, March 20(4), 499-514.

[3] Muc A., Ulatowska A., Local fibre reinforcement of holes in composite multilayered plates, Comp. Struct. 2012, 94 1413-1419.

[4] Engels S., Becker W., Optimization of hole reinforcements by doublers, Struct. Multidisc. Optim. 2000, 20, 57-66.

[5] Abuelfoutouh N.M., Preliminary design of unstiffened composite shells, Proc. Symp. 7th Technical Conf. ASC 1993, 693-786.

[6] Muc A., The mechanics of fiber composite materials (in Polish), Księgarnia Akademicka, Kraków 2003.

[7] Madenci E., Guven I., The Finite Element Method and Application in Engineering Using ANSYS, Springer Science + Business Media 2006.

[8] Muc A., Design of composite structures under cyclic loads, Comp. & Struct. 2000, 76, 211-218.

[9] Muc A., Kędziora P., A fuzzy set analysis for a fracture and fatigue damage response of composite materials, Comp. Struct. 2001, 54, 283-287.

[10] Muc A., A fuzzy set approach to interlaminar cracks simulation problems, Int. J. Fatigue 2002, 24, 419-427.