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Nandure Narayan Rao[1]*, Sirapu Sirisha[1], Pavuluri M.V. Rao[1], Sujith Bobba[2]**

[1]Department of Mechanical Engineering, Vignan's Foundation for Science Technology and Research (Deemed to be university), Guntur, India
[2]Department of Applied Engineering, Vignan's Foundation for Science Technology and Research (Deemed to be university), Guntur, India
** Corresponding author. E-mail: nandurerao@gmail.com, **co-Corresponding author. E-mail: sirishasirapu0352@gmail.com*

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EFFECT OF BOUNDARY CONDITIONS AND METAL ALLOY LAYER ON NATURAL FREQUENCIES OF FIBRE METAL LAMINATES

The natural frequencies and mode shapes of fibre metal laminates (FMLs) were numerically investigated and validated using commercially available finite element analysis software (ANSYS). Various grades of GLARE and FML were considered for free vibration analysis, and the effect of the central metal layer and aspect ratios on the frequencies were analysed for simply supported, clamped edge conditions. The obtained fundamental frequencies, natural frequencies and mode shapes comply with the available literature. The effect of the outermost metal layer on the natural frequencies was also investigated for various combinations of edge conditions. The obtained results indicate that there is a significant effect of the central and outermost metal layer on the natural frequencies, irrespective of the edge conditions and aspect (width/length) ratios.

Keywords: edge conditions, FEM, fibre metal laminate, natural frequencies, numerical method

INTRODUCTION

Composite materials have received a great deal of scientific interest during the last three decades. After World War II, the aircraft sector started employing composite materials commercially, particularly for military uses. Weight was significantly decreased because of the creation of composite materials and the use of those materials in structural design [1, 2]. These materials are preferred over conventional metallic alloys where high strength and stiffness-to-weight ratios are concerned; additionally, they have high specific stiffness, excellent corrosion resistance and fatigue resistance [3, 4]. With all these advantages, composite materials have gained widespread application in the aircraft industry.

The most important objective of the aviation industry in the late 1950s was to enhance the crack propagation properties of materials. Commercially available materials like aluminium alloys and fibre-reinforced composites have a few disadvantages like poor fatigue strength in the case of aluminium alloys and poor fracture toughness and residual strength regarding composites materials [5-7]. By taking advantage of the outstanding fatigue and fracture characteristics of fibre-reinforced composite materials and combining these properties with the durability and plasticity of metals, researchers at Delft University of Technology (DUT) found that fatigue crack growth can be decreased in adhesively bonded sheet materials if they are made by laminating thin metal alloy sheets with fibre reinforced composite layers instead of using one thick monolithic sheet [6]. Based upon their research, in 1978 the first fibre metal laminate (FML), ARALL, was introduced by DUT in their Aerospace Engineering facility [7, 8].

This new hybrid material was composed of high-strength thin aluminium plates with 0.2±0.4 mm thick composite prepregs. These thin metal sheets were bound together with fibre-reinforced adhesive matrices [9]. Due to their numerous additional benefits, FMLs offer better machinability, formability, fire, corrosion resistance and impact resistance than most thermoset composites. They also have a high specific strength, good resistance to fatigue crack growth, and lower density when compared to conventional aluminium alloys. FMLs also have superior fatigue and fracture characteristics to fibre-reinforced composites, as well as ductility and durability to aluminium alloys. Depending on the fibres, FMLs graded as ARALL are based on aramid fibres, whereas GLARE is based on high-strength glass fibres. In recent years, the aerospace industry has become more interested in another class – Ti-FMLs for long-range supersonic transportation [9].

With the increase in the usage of FMLs in aircraft structures, many researchers have investigated the dynamic response of FMLs subjected to low-velocity impact [10]. However, the dynamics properties of FMLs have not been studied extensively for structural applications. The identification of natural frequencies and the corresponding mode shapes of FMLs play a major role in predicting resonance when they are subjected to variable amplitudes.

During the last two decades, the free vibration of isotropic and composite plates has been studied extensively and various methods have been proposed to investigate the natural frequencies of plates. A brief literature review is presented here to identify the available methods to evaluate the natural frequencies of composite plates. Classical lamination plate theory (CLPT) and first-order shear deformation theory (FSDT) have been widely used to analyse composite plates. In CLPT, the effect of transverse shear deformation is neglected and was found to be ineffective to analyse thick composite plates as it underestimates the deformation and overestimates the natural frequencies [11]. On the other hand, FSDT is not capable of properly constraining all the displacements for clamped plates. To overcome the aforementioned limitations of CLPT and FSDT, higher-order shear deformation theory was developed to better analyse the bending, buckling and vibration of laminated composite plates [11-13]. Bassily et al. [14] used the Ritz method to estimate the natural frequencies of isotropic rectangular plates of various combinations of fixed, free, or simply supported boundary conditions. The results show that the fundamental frequencies slightly vary with the change in the side ratio, and it was observed that by reversing the direction of the end shearing force for any mode, the frequencies of the rectangular plate are unaffected Reddy et al. [11] developed analytical and finite element solutions using CLT, FSDT and third-order laminate theory to determine the fundamental frequencies for various aspect ratios of a cross-ply rectangular composite plate. The results show that shear deformation theory accurately predicts the natural frequencies, whereas CLT overestimates the same. Numayr et al. [15] developed a finite difference method to study the free vibration of composite plates. It was concluded that rotary inertia has a negligible influence on the natural frequencies, whereas a significant influence of shear deformation was reported for higher modes compared to lower modes. Palardy et al. [16] analytically investigated the effect of shear deformation and rotary inertia on fundamental frequencies and studied the vibration characteristics of graphite-epoxy symmetrical laminated cross-ply plates. Ju et al. [17] developed a finite element formulation using Mindlin plate theory for the analysis of free vibration. The numerical results indicate that the modal parameters of delaminated plates not only depend on the size of delamination, but also the edge conditions and mode number. Harras et al. [18] developed a theoretical model based on Hamilton's principle and used spectral analysis to examine the nonlinear free vibration of a hybrid GLARE-3 clamped rectangular plate, and good agreement was reported between the experimental and theoretically calculated natural frequencies. Xu et al. [19] considered the amplitude dependencies to study the nonlinear vibration of FMLs and concluded that with the increase in the excitation amplitude, the natural frequencies dropped steadily due to stiffness softening behaviour. Ghasemi and Mohandes [20] developed modified couple stress theory to study the frequencies of micro and nano FML cylindrical shells for various boundary conditions. The obtained results indicate that the frequencies of an ARALL micro cylinder are greater compared to CARALL and GLARE and the frequencies of a micro FML cylindrical shell decrease with the increase in the material length scale parameter. Pushparaj and Suresha [21] numerically obtained the natural frequencies of cantilever glass and carbon fibre-reinforced polymer composite plates using ANSYS. It was concluded that the orientation of the outermost layer and hybridization has a significant influence compared to the volume fraction of fibre and change in the matrix material on natural frequencies.

From the literature, it was observed that very limited information is available on the effect of the metal alloy layer and edge conditions on natural frequencies when FMLs are used as rectangular plates. The literature reveals that even though investigators have proposed various methods to investigate natural frequencies, which have limitations such as underestimating deformation and overestimating natural frequencies. In the present work, a numerical approximation was used to investigate the natural frequencies of a rectangular plate regarding various aspect ratios (width/length) and the obtained results are validated by using the commercially available finite element software ANSYS. The developed procedure is first validated by evaluating the results presented in reference [18]. The method is further extended to study (a) the effect of the central metal layer on natural frequencies, (b) the variation in natural frequencies for various boundary conditions of width/length ratios, (c) the effect of the outermost metal layer on the natural frequencies for clamped boundary conditions, and the results are presented.

GOVERNING EQUATION

Based on the assumptions of classical laminate plate theory (CLPT) and neglecting the effect of the thermal load, the governing differential equation for an unforced orthotropic plate is given as in [22].

$$D_{11}\frac{\partial^4 w_0}{\partial x^4} + 4D_{12} + 2D_{66}\frac{\partial^4 w_0}{\partial x^2 \partial y^2} + D_{22}\frac{\partial^4 w_0}{\partial y^4} + I_o\frac{\partial^2 w_0}{\partial t^2} + + I_2\left(\frac{\partial^4 w_0}{\partial t^2 \partial x^2} + \frac{\partial^4 w_0}{\partial t^2 \partial y^2}\right) = 0 \tag{1}$$

where: $I_0 = \sum_{k=1}^{L} \rho_0^{(k)}(z_{k+1} - z_k)$,

$$I_2 = \frac{1}{3}\sum_{k=1}^{L} \rho_0^{(k)}(z_{k+1}^3 - z_k^3) \tag{2}$$

Ritz method to investigate free vibration

Assuming the solution of differential Eq. (1) to be periodic, the solution can be given as:

$$w_0 = \omega(x, y)e^{(i\omega t)} \tag{3}$$

Substituting Eq. (3) in Eq. (1), the nontrivial solution of ω is given as:

$$D_{11}\frac{\partial^4 w_0}{\partial x^4} + 4(D_{12} + 2D_{66})\frac{\partial^4 w_0}{\partial x^2 \partial y^2} + \\ + D_{22}\frac{\partial^4 w_0}{\partial y^4} - \omega^2\left[I_0 w - I_2\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right)\right] = 0 \tag{4}$$

The weak form of Eq. (4) for natural vibration is expressed as

$$\int_0^b\int_0^a\left\{D_{11}\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 \delta w}{\partial x^2} + D_{12}\left(\frac{\partial^2 w}{\partial y^2}\frac{\partial^2 \delta w}{\partial x^2} + \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 \delta w}{\partial y^2}\right) + \\ + 4D_{66}\frac{\partial^2 w}{\partial x \partial y}\frac{\partial^2 \delta w}{\partial x \partial y} + 4D_{22}\frac{\partial^2 w}{\partial y^2}\frac{\partial^2 w}{\partial^2 y} - \omega^2[I_0 w \delta w + \\ + I_2\left(\frac{\partial w}{\partial x}\frac{\partial \delta w}{\partial x} + \frac{\partial w}{\partial y}\frac{\partial \delta w}{\partial y}\right)]\right\}dxdy \tag{5}$$

considering the Ritz method of solution and using an N-parameter Ritz solution results in

$$w(x,y) \approx \sum_{j=1}^{N} c_j \psi(x,y) \tag{6}$$

From Eq. (5) we obtain

$$([R] - \omega^2[b])\{c\} = \{0\} \tag{7}$$

From Eq. (6) for a rectangular plate, the Ritz approximation function can be expressed in the form of

$$w(x,y) \approx W_{mn}(x,y) = \sum_{i=1}^{N}\sum_{j=1}^{M} c_{ij} X_i(x) Y_j(y) \tag{8}$$

where X_i and Y_i are the functions selected in such a way that they satisfy the geometric boundary conditions. The coefficients of $R_{(ij)(kl)} of\ [R]$ and $B_{(ij)(kl)} of\ [B]$ are obtained as

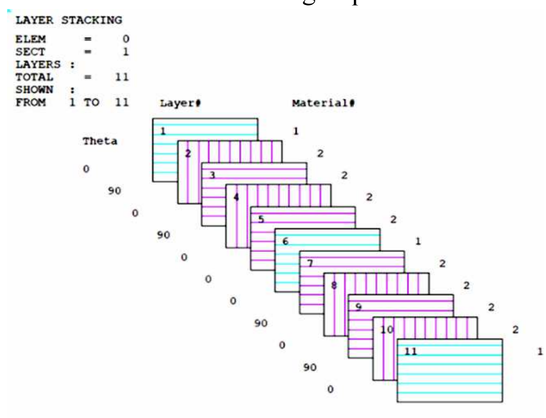
$$B_{(ij),(kl)} = \int_0^b\int_0^a\left[I_0 X_i X_k Y_j Y_l + \\ + I_2\left(Y_j Y_l \frac{dX_i}{dx}\frac{dX_k}{dx} + X_i X_k \frac{dY_j}{dy}\frac{dY_l}{dy}\right)\right]dxdy \tag{9}$$

$$R_{(ij),(kl)} = \int_0^b\int_0^a\left[D_{11}\frac{d^2X_i}{dx^2}\frac{d^2X_k}{dx^2} + \\ + D_{22}X_i X_k\frac{d^2Y_j}{dy^2}\frac{d^2Y_l}{dy^2} + D_{12}\left(X_i Y_l\frac{d^2X_k}{dx^2}\frac{d^2Y_j}{dy^2} + \\ + X_k Y_j\frac{d^2X_i}{dx^2}\frac{d^2Y_l}{dy^2}\right) + 4D_{66}\left(\frac{dX_i}{dx}\frac{dX_k}{dx}\frac{dY_j}{dy}\frac{dY_l}{dy}\right)\right]dxdy \tag{10}$$

FINITE ELEMENT METHOD

FEM is a numerical technique to solve approximate solutions of differential equations that may arise when solving engineering applications. FEM can characterize complicated geometrical domains and the application of physical boundary conditions, which made this method a popular tool for engineering design and analysis. In the present work, the multipurpose commercially available finite element software ANSYS is used to analyse the natural frequencies of fibre metal laminated plates. A 3D finite element rectangular plate using SHELL281 modelled in ANSYS is shown in Figure 1. The stacking sequence and the rectangular FML model in ANSYS are shown in Figure 1a and 1b, respectively. The corresponding SSSS and CCCC edge conditions used in the analysis are presented in Figure 2a and 2b, respectively. To obtain the natural frequencies, the complete procedure is coded in ANSYS Mechanical APDL.

a) Fibre metal laminate stacking sequences



b) FML model in ANSYS

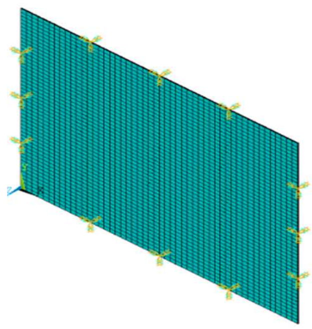
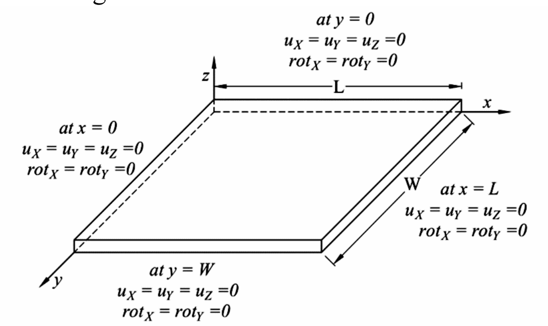


Fig. 1. Finite modelling of rectangular FML

a) SSSS edge conditions



b) CCCC edge conditions

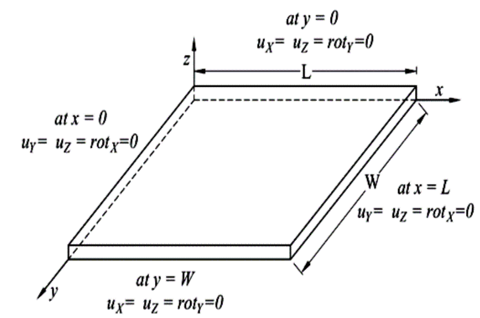


Fig. 2. Rectangular plate with edge conditions

NUMERICAL RESULTS AND DISCUSSION

The entire procedure of evaluating the natural frequencies is computerized by using the MATLAB routine. The natural frequencies of a laminated plate are obtained by means of the present method represented in the form of non-dimensional parameter $\bar{\omega} = \omega(b^2/\pi^2)\sqrt{\rho h/D_{22}}$. This method is further extended to study the natural frequencies of GLARE and metal-alloy-based FMLs. The developed method is validated by considering various configurations of composite laminates available in the literature.

Case 1: Effect of material properties on fundamental frequencies

Carbon reinforced, glass reinforced, and aramid reinforced composite laminates were selected to study the effect of E_{11}/E_{22} on the non-dimensional fundamental frequencies in relation to the width-to-length ratio. The composite material properties are presented in Table 1.

TABLE 1. **Material properties of composite materials [20]**

Material	Properties				
	E_{11} [GPa]	E_{22} [GPa]	ν_{12}	G_{12} [GPa]	ρ [kg/m^3]
Aramid epoxy	76	5.5	0.34	2.3	1460
Glass epoxy	38.6	8.27	0.26	4.14	1800
Carbon epoxy	181	10.3	0.28	10.7	1600

The results obtained from the analysis are presented in Figure 3. It can be observed that non-dimensional fundamental frequencies $\bar{\omega}$ of the carbon reinforced composite are higher compared to the other two composite materials because of the higher elasticity modulus of the reinforced carbon fibres. With the increase in width-to-length ratio $S = b/a$, the frequencies of the rectangular plate decline, and after a width-to-length ratio $S > 3$, all the plates have the same frequencies.

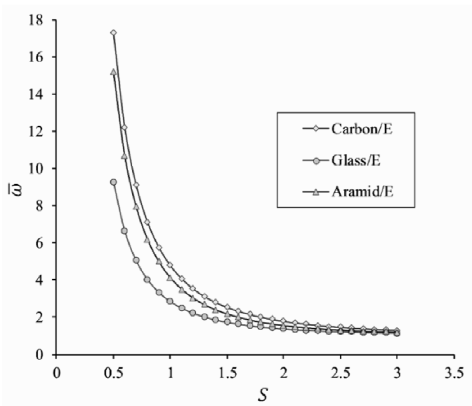


Fig. 3. Variation in non-dimensional fundamental frequencies $\bar{\omega}$ to aspect ratio

VALIDATION OF NUMERICAL METHOD AND FINITE ELEMENT ANALYSIS

Case 2: Vibration of GLARE 3(3/2 lay-up) CCCC plate

A CCCC rectangular plate of GLARE 3 (3/2 lay-up) was considered to validate the developed numerical methods and simulation results obtained from ANSYS. The obtained first five natural frequencies as well as mode shapes are compared with the references [17, 18] and the GLARE 3 material properties used in the analysis are presented in Table 3.

TABLE. 2. **Natural frequencies and mode shapes of CCCC rectangular plate GLARE 3 (3/2)**

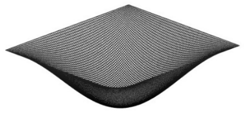
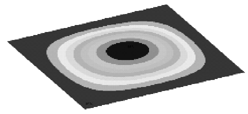
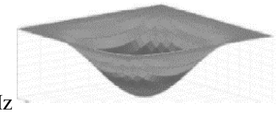
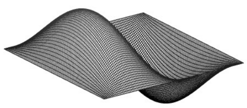
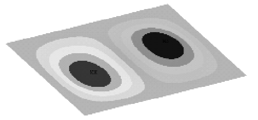
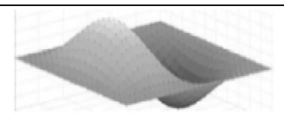
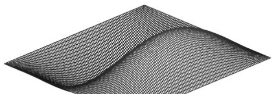
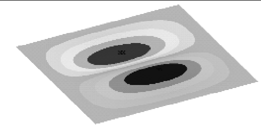
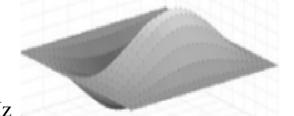
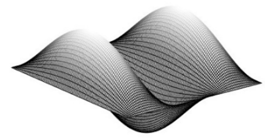
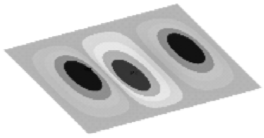
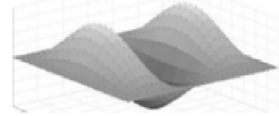
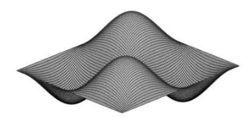
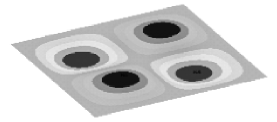
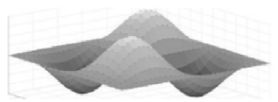
Mode	Mode Shapes		
	Present numerical	**Present FEM(ANSYS)**	**Reference [18]**
1	 105.94 Hz	 104.51 Hz	 105.5Hz
2	 166.61 Hz	 161.03 Hz	 161.9Hz
3	 256.29 Hz	 256.16 Hz	 257.4Hz
4	 264.88 Hz	 257.04 Hz	 258.6Hz
5	 318.01 Hz	 307 Hz	 309.7Hz

TABLE 3. **Material properties of GLARE 3 [17, 18]**

Material	Properties							
	E_{11} [GPa]	E_2 [GPa]	ν_{12}	G_{12} [GPa]	ρ [kg/m^3]	Length [mm]	Width [mm]	T [mm]
Glass fibre	31.17	9.412	0.098	5.548	2.000	450	300	0.125
Aluminium	72.39	--	0.33	41.5	2.700	450	300	0.3

It can be observed from Table 2 that the natural frequencies and mode shapes, respectively, of GLARE 3 obtained by means of calculation are the same as those obtained in [18], which validates the correctness of the proposed model.

Case 3: Effect of central metal layer on natural frequencies of rectangular SSSS plate

In this case, GLARE 3 (3/2 lay-up) was chosen to analyse the effect of the central metal-alloy layer on natural frequencies. The results obtained from the numerical method and ANSYS are presented in Table 4 and the material properties are given in Table 3.

TABLE 4. **Natural frequencies (Hz) and mode shapes of SSSS cross-ply laminate for, L = 450 mm, W = 300 mm**

	Modes					
	1	2	3	4	5	6
$[Al/0/90/Al/90/0/Al]$						
Numerical	58.17	112.40	172.80	198.42	232.69	317.38
FEM (ANSYS)	54.89	105.48	169.75	190.39	219.50	303.38
$[Al/0/90/90/0/Al]$						
Numerical	42.09	81.36	124.79	143.46	168.38	235.65
FEM (ANSYS)	45.17	86.86	139.34	156.52	180.65	249.89

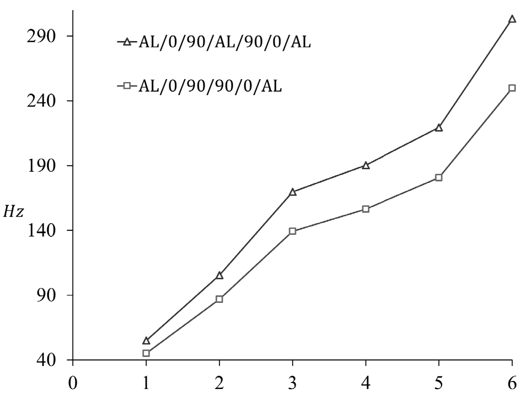


Fig. 4. Variation in natural frequencies in relation to mode

The deviation of the natural frequencies of [*Al*/0/90/*Al*/90/0/*Al*] and [*Al*/0/90/90/0/*Al*] from the mode are presented in Figure 4. From the graph, it can be observed that there is less variation in the fundamental frequencies for both the configurations, whereas with the increase in the mode, the difference in the natural frequencies between these two configurations increases, and also the frequencies of [*Al*/0/90/*Al*/90/0/*Al*] having a central metal-alloy layer are more than the [*Al*/0/90/90/0/*Al*] without a central metal-alloy layer. The dominance of the central metal-alloy layer can be observed with the increase in the mode at the highest natural frequencies.

Case 4: Effect of boundary conditions on natural frequencies of GLARE 3-(3/2) laminate

[*Al*/0/90/*Al*/90/0/*Al*] was selected to analyse the effect of the boundary conditions and width-to-length ratio on the fundamental frequencies. The calculated non-dimensional fundamental frequencies are represented in Table 5. For all the types of boundary conditions, the fundamental frequencies decrease with the increase in the width-to-length ratio.

TABLE 5. **Fundamental frequencies (Hz) of Al-GFRP laminate plate for various edge conditions**

width/length		SSSS	SSCC	SSSC	SSSF	SSFC
[*Al*/0/90/*Al*/90/0/*Al*]						
0.5	Numerical	192.49	201.41	198.25	162.74	165.21
	FEM	190.39	211.23	199.35	159.23	161.08
1.0	Numerical	77.25	113.22	93.28	45.52	49.66
	FEM	75.88	111.78	91.09	44.82	48.59
1.5	Numerical	55.29	97.58	75.25	24.56	30.28
	FEM	54.89	96.93	73.01	23.20	28.79
2.0	Numerical	48.44	93.21	69.54	16.32	22.77
	FEM	47.61	92.30	67.07	15.27	21.73
2.5	Numerical	45.21	91.11	66.35	13.21	20.54
	FEM	44.26	90.26	64.42	11.34	18.71
3.0	Numerical	43.85	91.11	65.21	11.10	19.55
	FEM	42.45	89.22	63.01	9.04	17.12

Case 5: Effect of metal-alloy layer on natural frequencies of cross-ply laminates with clamped edges

GLARE 5 (3/2)-0.4 (AL 2024-T3) and GLARE 5-(3/2)-0.5 Mg (Mg AZ31BH24) are analysed for the effect of the metal layer on the natural frequencies for various aspect ratios. Composite prepreg layers are made of FM94/S2 (FM94 epoxy adhesive film, S2 glass fibre, and UD). The material properties of the aluminium, magnesium alloy and FM94/S2 are presented in Table 6. The variations in the fundamental frequencies in relation to the width/length ratio are shown in Figure 5.

TABLE 6. **Material properties [20]**

Material	Properties					
	E_{11} [GPa]	E_{22} [GPa]	ν_{12}	G_{12} [GPa]	ρ [kg/m^3]	t [mm]
FM94/S2	53.7	9.1	0.29	3.40	1.960	0.127
Al 2024-T3	72.4	-	0.33	27.20	2.770	0.404
Mg AZ31B-H24	45.0	-	0.35	16.67	1.780	0.508

From Figure 5, it can be observed that the natural frequencies of Al-GFRP FML are greater compared to Mg-GFRP FMLs for the same width-to-length ratios. With the increase in the width-to-length ratio, the natural frequencies decrease. Beyond a width-to-length ratio of 2, both the FMLs have the same natural frequencies.

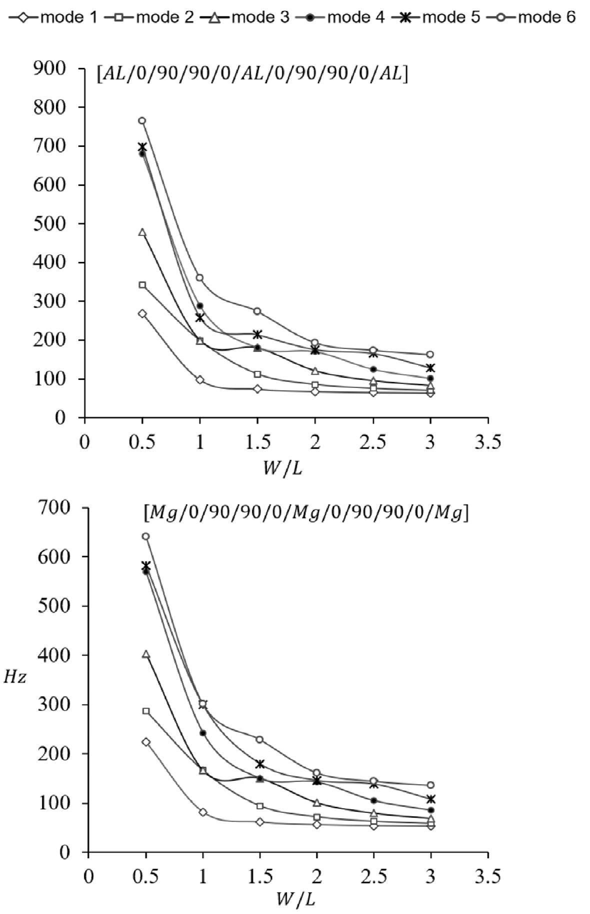


Fig. 5. Variation in fundamental frequencies in relation to W/L ratio

CONCLUSION

A numerical method was developed to study and validate the free vibration of rectangular plates for various aspect ratios, using commercially available finite element software. The results were obtained for various aspect ratios and the influence of the central and outermost metal layers on the natural frequencies for different boundary conditions were discussed. From the results, the following conclusions were drawn:

a) The natural frequencies of FMLs depend on the central and outermost metal layers for all types of edge conditions.
b) The central metal layer does not have much effect on the fundamental frequency, but at higher modes, the natural frequencies of the FMLs with a central metal layer are greater compared to FMLs without a central layer.
c) The natural frequencies decrease with the increase in the aspect ratios; after a certain aspect ratio all the configurations of FMLs have approximately the same natural frequencies.

REFERENCES

[1] Cocchieri E., Almeida R., José S., Paulo S., A Review on the development and properties of continuous fiber/epoxy/aluminium hybrid composites for aircraft structures, Mater. Res. 2006, 9(3), 247-256.

[2] Rao N.N., Rao P.M.V., Kumar S., A numerical approach to estimate first ply failure of fibre metal laminate. Revue des Composites et des Matériaux Avancés – Journal of Composite and Advanced Materials 2021, 31(1), 33-39, DOI: 10.18280/rcma.310105.

[3] Morinière F.D., Alderliesten R.C., Benedictus R., Modelling of impact damage and dynamics in fibre-metal laminates – A review, Int. J. Impact Eng. 2014, 67, 27-38, DOI: 10.1016/j.ijimpeng.2014.01.004.

[4] Krishnakumar S., Fiber metal laminates – the synthesis of metals and composites, Material and Manufacturing Process 1994, 9(2), 295-354, DOI: 10.1080/10426919408934905.

[5] Vogelesang L.B., Vlot A., Development of fibre metal laminates for advanced aerospace structures, J. Mater. Process. Technol. 2000, 103(1), 1-5, DOI: 10.1016/S0924-0136(00)00411-8.

[6] Asundi A., Choi A.Y.N., Materials processing technology fiber metal laminates: an advanced material for future aircraft, J. Mater. Process. Technol. 1997, 63(1-3), 384-394, DOI: 10.1016/S0924-0136(96)02652-0.

[7] Vlot A., Vogelesang L.B., De Vries T.J., Towards application of fibre metal laminates in large aircraft, Aircr. Eng. Aerosp. Technol. 1999, 71(6), 558-570.

[8] Narayan Rao N., Rao P.M.V., First ply failure analysis of rectangular fiber metal laminated composite plates subjected to uniformly distributed loads, J. Fail. Anal. and Preven. 2019, 19(6), 1683-1690, DOI: 10.1007/s11668-019-00768-x.

[9] Sinmazçelik T., Avcu E., Bora M.Ö., Çoban O., A review: Fibre metal laminates, background, bonding types and applied test methods, Materials and Design 2008, (32)7, 3671--3685, DOI: 10.1016/j.matdes.2011.03.011.

[10] Bieniaś J., Jakubczak P., Low velocity impact resistance of aluminium/carbon-epoxy fiber metal laminates, Compos. Theory Pract. 2012, 12, 193-197.

[11] Reddy J.N., Khdeir A.A., Buckling and vibration of laminated composite plates using various plate theories, AIAA J. 1989, 27(12), 1808-1817.

[12] Sayyad A.S., Ghugal Y.M., On the free vibration analysis of laminated composite and sandwich plates: A review of recent literature with some numerical results, Compos. Struct.

2015, (129), 177-201, DOI: 10.1016/j.compstruct.2015.04.007.

[13] Zhang Y.X., Yang C.H., Recent developments in finite element analysis for laminated composite plates, Compos. Struct. 2009, 88(1), 147-157, DOI: 10.1016/j.compstruct.2008.02.014.

[14] Bassily S.F., Dickinson S.M., Buckling and lateral vibration of rectangular plates subject to inplane loads – a Ritz approach, J. Sound Vib. 1972, 24(2), 219-239, DOI: 10.1016/0022-460X(72)90951-0.

[15] Numayr K.S., Haddad R.H., Haddad M.A., Free vibration of composite plates using the finite difference method, Thin--Walled Struct. 2004, 42(3), 399-414, DOI: 10.1016/j.tws.2003.07.001.

[16] Palardy R.F., Palazotto A.N., Buckling and vibration of composite plates using the Levy method, Compos. Struct. 1990, 14(1), 61-86, DOI: 10.1016/0263-8223(90)90059-N.

[17] Ju F., Lee H.P., Lee K.H., Finite element analysis of free vibration of delaminated composite plates, Compos. Eng. 1995, 5(2), 195-209, DOI: 10.1016/0961-9526(95)90713-L.

[18] Harras B., Benamar R., White R.G., Experimental and theoretical investigation of the linear and non-linear dynamic behaviour of a glare 3 hybrid composite panel, J. Sound Vib. 2002, 252(2), 281-315, DOI: 10.1006/jsvi.2001.3962.

[19] Xu Z., Jiang Gao Z., Qi Zhao S., Feng Zhang Y., Chun Wen B., A nonlinear vibration model of fiber metal laminated thin plate with amplitude dependent property, Appl. Acoust. 2020, 164(1-14), DOI: 10.1016/j.apacoust.2020.107268.

[20] Ghasemi A.R., Mohandes M., Free vibration analysis of micro and nano fiber-metal laminates circular cylindrical shells based on modified couple stress theory, Mech. Adv. Mater. Struct. 2020, 27(1), 43-54, DOI: 10.1080/15376494.2018.1472337.

[21] Pushparaj P., Suresha B., Free vibration analysis of laminated composite plates using finite element method, Polym. Polym. Compos. 2016, 24(7), 529-538, DOI: 10.1177%2F096739111602400712.

[22] Reddy J.N., Mechanics of Laminated Composite Plates and Shells: Theory and Analysis, 1st ed., CRC Press, 2003.

