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STRESS AND FAILURE ANALYSIS OF COMPOSITE PLATES WITH CIRCULAR HOLE SUBJECTED TO SHEAR LOADING. PART 1: ANALYTICAL AND FINITE ELEMENT FORMULATION

This current paper, which is the first part of two parts of a complete article, presents the theoretical and finite element formulation developed and proposed by the authors to obtain the stress concentration factors (SCFs) and the first ply failure (FPF) loads of composite laminated plates. The numerical studies are performed using a quadrilateral finite element of four nodes with thirty-two degrees of freedom. The present finite element was previously developed by the authors to study the bending and buckling of composite plates. The present finite element is a combination of two finite elements. The first one is a linear isoparametric membrane element, and the second one is a high-precision rectangular Hermitian element. In the second part of the paper, several examples will be considered to demonstrate and affirm the accuracy and the performance of the present element, as well as highlight the effect of some parameters on the stress distribution. The FPF strengths and their locations in laminated plates with and without holes are calculated by adapting the Hashin-Rotem, Tsai-Hill, and Tsai-Wu failure theories.

Keywords: anisotropic plates, stress concentration, circular cutout, failure criterion, shear loading

INTRODUCTION

Composite materials have widespread applications in aerospace and other industries where weight reduction and directional properties are the main criteria [1]. Circular cutouts are unavoidable in these materials to satisfy the needs of the design. Cutouts change the mechanical behavior of structures and produce a highly undesirable stress concentration around these cutouts [2-6]. The accurate design of laminated plates with notches requires the stress distribution to be determined by means of appropriate methods [7]. Stress analysis has received much attention from many researchers, who have carried out their studies using different analytical and numerical approaches [8-11]. Significant progress in the analytical solution for anisotropic structures with cutouts was achieved by Lekhnitskii [12] and Savin [13]. These methods essentially generalize the Muskhelishvili [14] method for resolving two-dimensional problems of isotropic materials with complex stress analysis. Some recent references for the contemporary application of these analytical methods can be found in [15-19].

Recently, Dharmin et al. [20] and Nagpal et al. [21] published review papers regarding recent analytical methods and numerical techniques for the stress analysis of composite laminated plates with cutouts. As a result, only few investigations on laminated plates under shear loading were found in comparison to numerical investigations on plates under tensile loading. Thus, most research studies have been performed using analytical approaches based on Lekhnitskii [12] or Savin [13], whereas the only in-depth research work based on the theoretical elasticity of Green and Zerna [22] was carried out by Arslan [23]. In his study, a very simple analytical solution to determine the stress distribution in single-layer plates is proposed. Arslan [23] studied the fiber orientation angle effect on the stress values located in the vicinity of circular cutouts in single-layers, and their failure loads using Tsai-Hill [24] and Hencky-von Mises failure criteria. On the other hand, to determine the stress field in a composite material, the finite element method (FEM) can be considered a very powerful numerical tool to analyze the stress distribution in structures with cutouts of complicated geometry. Some recent applications and references for determining the stress concentration employing FEM can be found in [6, 25-41].

The current work calculates the stress concentration numerically by means of the developed finite element. In the first section of the present paper, the mathematical formulation of the developed element is presented in detail. Based on the classical plate theory, the finite element can be considered as a combination of two finite elements. The first one is a linear isoparametric membrane element. The second finite element is a high-precision Hermitian element [42]. The first section is followed by an analytical review of some selected approaches, which can be used to determine the stress field in anisotropic plates under shear loading. In the second part of the paper, several new numerical examples will be considered to affirm and reveal the accuracy and the robustness of the present finite element to solve these kinds of problems. On the other hand, the effects of different parameters such as the fiber orientation, stacking sequence and the degree of anisotropy are studied. In the last section of the present paper, the mathematical formulations of other failure theories are presented in detail to calculate the failure strength of the laminated plates with and without cutouts.

THEORETICAL FORMULATION

Based on Kirchhoff's assumptions, the displacement field, according to the classical laminated plate theory, is given by the following:

$$\begin{aligned} U(x,y,z) &= u_0(x,y) - z\frac{\partial w}{\partial x} \\ V(x,y,z) &= v_0(x,y) - z\frac{\partial w}{\partial y} \\ W(x,y,z) &= w_0(x,y) \end{aligned} \tag{1}$$

where u_0, v_0 and w_0 are respectively, in-plane and transverse displacement components at the mid-plane of the plate.

Strain-displacement relations

The strain-displacement relations, including large deformations, can be determined as:

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u}{\partial x} + \dfrac{1}{2}\left(\dfrac{\partial w}{\partial x}\right)^2 \\ \dfrac{\partial v}{\partial y} + \dfrac{1}{2}\left(\dfrac{\partial w}{\partial y}\right)^2 \\ \dfrac{\partial v}{\partial x} + \dfrac{\partial u}{\partial y} + \dfrac{\partial w}{\partial x}\dfrac{\partial w}{\partial y} \end{Bmatrix} + z\begin{Bmatrix} -\dfrac{\partial^2 w}{\partial x^2} \\ -\dfrac{\partial^2 w}{\partial y^2} \\ -2\dfrac{\partial^2 w}{\partial x \partial y} \end{Bmatrix} = \underbrace{\left\{\varepsilon^0_{\mathrm{L}} + \varepsilon^0_{\mathrm{NL}}\right\}}_{\{\varepsilon\}} + z\{k\} \tag{2}$$

Stress-strain relationships

By adopting the classical laminate plate theory, membrane N and moments M resultants are related to the mid-surface strains and curvatures k by:

$$\begin{bmatrix} N \\ M \end{bmatrix} = \begin{bmatrix} [A] & [B] \\ [B] & [D] \end{bmatrix} \begin{Bmatrix} \varepsilon^0 \\ k \end{Bmatrix} \tag{3}$$

where $[A]$, $[B]$ and $[D]$ are the extensional, coupling and bending rigidity matrix, respectively, which can be defined by:

$$\{A,B,D\}^{\mathrm{T}} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \left[\overline{Q}_{ij}\right]_k (1,z,z^2)dz \tag{4}$$

with $\overline{Q}_{ij}$ are the coefficients of elasticity of a layer in the global coordinate system (x, y, z) of the laminate forming an angle θ with the local coordinate system (1, 2, 3) (Fig. 1).

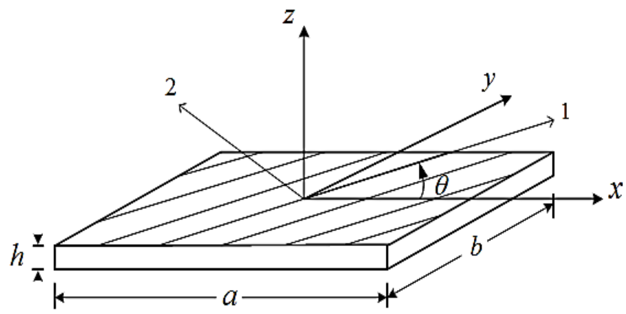

Fig. 1. Geometry and coordinate systems of rectangular laminated composite plate

FINITE ELEMENT FORMULATION

The proposed element is a combination of an isoperimetric membrane quadrilateral element and a first-order Hermitian rectangular plate element with a high degree of accuracy. The element has four nodes with eight degrees of freedom each.

The displacement state leads to thirty-two degrees of freedom per element with eight degrees of freedom per node, and the resulting displacement vector $\{q\}$ is:

$$\{q\} = \left\{ u_i, v_i, w_i, \frac{\partial w_i}{\partial x}, \frac{\partial w_i}{\partial y}, \frac{\partial^2 w_i}{\partial x \partial y}, \frac{\partial^2 w_i}{\partial x^2}, \frac{\partial^2 w_i}{\partial y^2} \right\}_{i=1,4} \tag{5}$$

As the element is a combination of an isoperimetric membrane element and a high-precision plate element of the Hermitian type, interpolation functions $N_i(\xi,\eta)$ of the coordinates and displacements through the element are given by:

$$\begin{aligned} x(\xi,\eta) &= \sum \left(N_i(\xi,\eta)x_i\right)_{i=1,4} \\ y(\xi,\eta) &= \sum \left(N_i(\xi,\eta)y_i\right)_{i=1,4} \\ u(\xi,\eta) &= \sum \left(N_i(\xi,\eta)u_i\right)_{i=1,4} \\ v(\xi,\eta) &= \sum \left(N_i(\xi,\eta)v_i\right)_{i=1,4} \\ N_i(\xi,\eta) &= \frac{1}{4}(1+\xi\xi_i)(1+\eta\eta_i) \end{aligned} \tag{6}$$

The transverse displacement of the element of reference is expressed as a product of one dimensional first order Hermitian interpolation polynomial:

$$w = H_{00} w_i + H_{10}\frac{\partial w}{\partial \xi} + H_{01}\frac{\partial w}{\partial \eta} + H_{11}\frac{\partial^2 w}{\partial \xi\, \partial \eta} \tag{7}$$

where:

$$\begin{aligned}
H_{00} &= \frac{1}{16}(\xi+\xi_0)^2(\xi\xi_0-2)(\eta+\eta_0)^2(\eta\eta_0-2)\\
H_{10} &= -\frac{1}{16}\xi_0(\xi+\xi_0)^2(\xi\xi_0-1)(\eta+\eta_0)^2(\eta\eta_0-2)\\
H_{01} &= \frac{1}{16}(\xi+\xi_0)^2(\xi\xi_0-2)(\eta+\eta_0)^2(\eta\eta_0-1)\\
H_{11} &= \frac{1}{16}\xi_0(\xi+\xi_0)^2(\xi\xi_0-1)(\eta+\eta_0)^2(\eta\eta_0-1)
\end{aligned} \tag{8}$$

The passage of the rectangular reference element to the real quadrilateral element requires the following transformations:

$$\begin{aligned}
\frac{\partial w}{\partial \xi} &= \frac{\partial w}{\partial x}\frac{\partial x}{\partial \xi} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial \xi}\\
\frac{\partial w}{\partial \eta} &= \frac{\partial w}{\partial x}\frac{\partial x}{\partial \eta} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial \eta}\\
\frac{\partial^2 w}{\partial \xi \partial \eta} &= \frac{\partial^2 w}{\partial x^2}\frac{\partial x}{\partial \xi}\frac{\partial x}{\partial \eta} + \frac{\partial^2 w}{\partial x \partial y}\left(\frac{\partial x}{\partial \xi}\frac{\partial y}{\partial \eta} + \frac{\partial x}{\partial \eta}\frac{\partial y}{\partial \xi}\right) +\\
&+ \frac{\partial^2 w}{\partial y^2}\frac{\partial y}{\partial \xi}\frac{\partial y}{\partial \eta} + \frac{\partial w}{\partial x}\frac{\partial^2 x}{\partial \xi\, \partial \eta} + \frac{\partial w}{\partial y}\frac{\partial^2 y}{\partial \xi\, \partial \eta}
\end{aligned} \tag{9}$$

Then, after transformation, the interpolation functions of the real element are written as:

$$w = L_w w + L_{\theta x}\frac{\partial w}{\partial x} + L_{\theta y}\frac{\partial w}{\partial y} + L_{\theta xy}\frac{\partial^2 w}{\partial x \partial y} + L_{\theta xx}\frac{\partial^2 w}{\partial x^2} + L_{\theta yy}\frac{\partial^2 w}{\partial y^2} \tag{10}$$

with:

$$\begin{aligned}
L_w &= H_{00}\\
L_{\theta x} &= H_{10}\frac{\partial x}{\partial \xi} + H_{01}\frac{\partial x}{\partial \eta} + H_{11}\frac{\partial^2 x}{\partial \xi\, \partial \eta}\\
L_{\theta y} &= H_{10}\frac{\partial y}{\partial \xi} + H_{01}\frac{\partial y}{\partial \eta} + H_{11}\frac{\partial^2 y}{\partial \xi\, \partial \eta}\\
L_{\theta xy} &= H_{11}\left(\frac{\partial x}{\partial \xi}\frac{\partial y}{\partial \eta} + \frac{\partial x}{\partial \eta}\frac{\partial y}{\partial \xi}\right)\\
L_{\theta xx} &= H_{11}\frac{\partial x}{\partial \xi}\frac{\partial x}{\partial \eta}\\
L_{\theta yy} &= H_{11}\frac{\partial y}{\partial \xi}\frac{\partial y}{\partial \eta}
\end{aligned} \tag{11}$$

The total potential energy of deformation of a plate subjected to a distributed transverse loading is given by:

$$\Pi = U + V' \tag{12}$$

Strain energy U of the element is given by:

$$U = \frac{1}{2}\int_{-\frac{a}{2}}^{\frac{a}{2}}\int_{-\frac{b}{2}}^{\frac{b}{2}}\begin{pmatrix}\{\varepsilon_L^0\}^T[A]\{\varepsilon_L^0\} + \{\varepsilon_L^0\}^T[B]\{k\} + \{k\}^T[B]\{\varepsilon_L^0\} + \\ \{k\}^T[D]\{k\} + \{\varepsilon_{NL}^0\}^T[N]\end{pmatrix} dxdy \tag{13}$$

The potential energy of a plate subjected to a distributed transverse loading is given by:

$$V' = -\int_{-\frac{a}{2}}^{\frac{a}{2}}\int_{-\frac{b}{2}}^{\frac{b}{2}} P\, w(x,y)\,dxdy \tag{14}$$

By introducing the interpolation polynomials into Eq. (13), strain energy U of the element becomes:

$$U = \frac{1}{2}\int_{-1}^{1}\int_{-1}^{1}\{q\}^T\begin{pmatrix}[S_\varepsilon]^T[A][S_\varepsilon] + [S_\varepsilon]^T[B][S_k]\\ +[S_k]^T[B][S_\varepsilon] + [S_k]^T[D][S_k]\end{pmatrix}\{q\}|J|\,d\xi d\eta \tag{15}$$

where

$$\begin{aligned}
\{\varepsilon_L^0\} &= [S_\varepsilon]\{q\}\\
\{k\} &= [S_k]\{q\}
\end{aligned} \tag{16}$$

In which $\{q\}$, $[S_\varepsilon]$ and $[S_k]$ are the resulting displacement vector of the element, the strain displacement matrix for membrane and bending, respectively. $|J|$ is the determinant of the Jacobian matrix. The equilibrium configuration is defined by minimizing the total potential energy, which means cancellation of the first variation:

$$\delta\Pi = \delta U + \delta V' = 0 \tag{17}$$

$$\begin{aligned}
&\int_{-1}^{1}\int_{-1}^{1}\{\partial q\}^T\begin{pmatrix}\{S_\varepsilon\}^T[A]\{S_\varepsilon\} + \{S_\varepsilon\}^T[B]\{S_k\}\\ +\{S_k\}^T[B]\{S_\varepsilon\} + \{S_k\}^T[D]\{S_\varepsilon\}\end{pmatrix}\{q\}|J|\,d\xi d\eta\\
&-\int_{-1}^{1}\int_{-1}^{1} P[L]\{\partial q\}|J|\,d\xi d\eta =
\end{aligned} \tag{18}$$

Equatin (18) allows the following equilibrium equation to be obtained:

$$[K_e]\{q\} = \{F_e\} \tag{19}$$

The expression of elementary stiffness matrix $[K_e]$ is:

$$[K_e] = \int_{-1}^{1}\int_{-1}^{1}\begin{pmatrix}\{S_\varepsilon\}^T[A]\{S_\varepsilon\} + \{S_\varepsilon\}^T[B]\{S_k\}\\ +\{S_k\}^T[B]\{S_\varepsilon\} + \{S_k\}^T[D]\{S_\varepsilon\}\end{pmatrix}|J|\,d\xi d\eta \tag{20}$$

And the elementary force vector is given as:

$$\left[F_e\right] = \int_{-1}^{1}\int_{-1}^{1} P\left[L\right]\left|J\right| d\xi d\eta \tag{21}$$

ANALYTICAL STRESS ANALYSIS

Analytical stress function for laminated plates under shear load

In the present investigation, use of the analytical stress function is proposed. This analytical stress function was suggested by Arslan [23] for determining the stress distribution in composite plates weakened by circular notches. A single layer plate under shear loading condition S is shown in Figure 2. The local coordinate system (1, 2) is associated with the fiber orientation angle, which forms angle θ with respect to the global coordinate system (x, y).

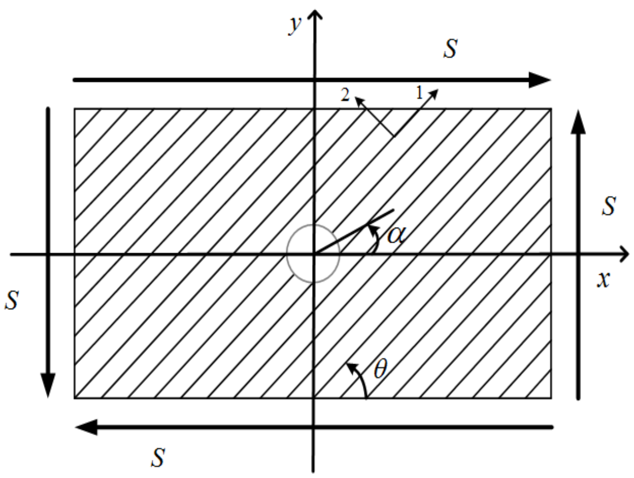

Fig. 2. A single composite layer subjected to shear load

Arslan [23] proposed a very accurate and simple stress function to determine the stress distribution around a circular notch. Using this stress function, the circumferential stress developed in the vicinity of the circular cutout can be calculated as:

$$\sigma_\alpha = \frac{\left[(N_1 - N_2)\sin 2\theta + N_3 \cos 2\theta\right]S}{(1+\gamma_1{}^2 - 2\gamma_1 \cos 2(\alpha-\theta))(1+\gamma_2{}^2 - 2\gamma_2 \cos 2(\alpha-\theta))}$$

where:

$$\begin{aligned} N_1 &= (1+\gamma_1)\cdot(1+\gamma_2)(1+\gamma_1+\gamma_2-\gamma_1\gamma_2-2\cos 2(\alpha-\theta)) \\ N_2 &= (1-\gamma_1)\cdot(1-\gamma_2)+(1-\gamma_1-\gamma_2-\gamma_1\gamma_2+2\cos 2(\alpha-\theta)) \\ N_3 &= 4(\gamma_1\gamma_2-1)\sin 2(\alpha-\theta) \end{aligned} \tag{22}$$

where γ_1 and γ_2 are defined as:

$$\gamma_1 = \frac{\left[\left(\frac{E_2}{2G_{12}} - \nu_{21}\right) + \left[\left(\frac{E_2}{2G_{12}} - \nu_{21}\right)^2 - \frac{E_2}{E_1}\right]^{1/2}\right]^{1/2} - 1}{\left[\left(\frac{E_2}{2G_{12}} - \nu_{21}\right) + \left[\left(\frac{E_2}{2G_{12}} - \nu_{21}\right)^2 - \frac{E_2}{E_1}\right]^{1/2}\right]^{1/2} + 1}$$

$$\gamma_2 = \frac{\left[\left(\frac{E_2}{2G_{12}} - \nu_{21}\right) - \left[\left(\frac{E_2}{2G_{12}} - \nu_{21}\right)^2 - \frac{E_2}{E_1}\right]^{1/2}\right]^{1/2} - 1}{\left[\left(\frac{E_2}{2G_{12}} - \nu_{21}\right) - \left[\left(\frac{E_2}{2G_{12}} - \nu_{21}\right)^2 - \frac{E_2}{E_1}\right]^{1/2}\right]^{1/2} + 1} \tag{23}$$

Remark: The stress function proposed by Arslan [23] can also be used to determine the stress field in isotropic plates subjected to shear loading. The stress distribution can be obtained by taking $\gamma_1 = \gamma_2 = 0$.

Remark: The stress function proposed by Arslan [23] is valid only if the analyzed layer is infinite (the notch size is very small compared to the plate dimension).

Failure strength

It is well known that one of the main objectives of calculating the stress in perforated plates is to predict the strength because the laminate strength will decrease considerably if the plate is weakened by a circular notch. In the present context, one can define the failure load as the maximum remote stress that the perforated laminate can carry without yielding or failure. It is well known that the strength in each layer depends on the fiber orientation, layer thickness and other parameters. Consequently, the failure strength is different in each layer. The minimum value of strength calculated in the vicinity of the notch can be considered as the failure load of that particular layer and the minimum strength of all the layers can be considered as the failure load of the whole laminate based on first ply failure theory. The failure theories that were used in this investigation are able to indicate any nonlinearity in the structure behavior or the occurrence of failure initiation, but they are not able to explain the failure mechanisms.

In isotropic plates, the failure around the holes usually depends on stress concentrations. However, in orthotropic plates with cutouts, and due to the coupling phenomenon presented in composite materials, failure will initiate at the notch edge not as a result of only the stress concentration, but instead due to the interaction of different stress components [11]. The present section will briefly present the mathematical formulation of various failure theories. To use these theories, the circumferential stress calculated using Eq. (22) should be transformed first to the local coordinate system and then they can be introduced into various failure criteria:

$$\begin{aligned}
\sigma_1 &= \sigma_\alpha \sin^2(\alpha-\theta) \\
\sigma_2 &= \sigma_\alpha \cos^2(\alpha-\theta) \\
\sigma_{12} &= -\sigma_\alpha \sin(\alpha-\theta)\cos(\alpha-\theta)
\end{aligned} \tag{24}$$

As was mentioned before, the failure strength of each ply in the laminated plate will be calculated and the minimum one will be considered as the failure load of the whole plate based on FPF theory. The failure load is obtained using the following theories.

Hashin-Rotem (H-R) criterion

According to the Hashin-Rotem theory [43], failure load σ_f of a unidirectional laminate can be calculated by means of the following equation:

$$\sigma_f^2 = \frac{1}{\left(\frac{\sigma_2}{\sigma}\right)^2 \frac{1}{Y^2} + \left(\frac{\sigma_{12}}{\sigma}\right)^2 \frac{1}{S'^2}} \tag{25}$$

Tsai-Hill (T-H) criterion

According to the Tsai-Hill theory [24], failure strength σ_f can be calculated using the following equation:

$$\sigma_f^2 = \frac{1}{\left(\frac{\sigma_1}{\sigma}\right)^2 \frac{1}{X^2} + \left(\frac{\sigma_2}{\sigma}\right)^2 \frac{1}{Y^2} + \left(\frac{\sigma_{12}}{\sigma}\right)^2 \frac{1}{S'^2} - \left(\frac{\sigma_1\sigma_2}{\sigma^2}\right)\frac{1}{X^2}} \tag{26}$$

Tsai-Wu (T-W) criterion

The Tsai-Wu criterion [44] can be written in the following form to calculate the failure load:

$$\sigma\, f_1\left(\frac{\sigma_1}{\sigma}\right) + \sigma\, f_2\left(\frac{\sigma_2}{\sigma}\right) + \sigma^2 f_{11}\left(\frac{\sigma_1}{\sigma}\right)^2 + \sigma^2 f_{22}\left(\frac{\sigma_2}{\sigma}\right)^2 + $$
$$+\sigma^2 f_{66}\left(\frac{\sigma_{12}}{\sigma}\right)^2 + 2\sigma^2 f_{12}\left(\frac{\sigma_1\sigma_2}{\sigma^2}\right) = 1 \tag{27}$$

By designating the value of remotely applied stress σ that causes failure as σ_f, then Eq. (27) can be rewritten as:

$$a\sigma_f^2 + b\sigma_f - 1 = 0 \tag{28}$$

$$\begin{aligned}
a &= f_{11}\left(\frac{\sigma_1}{\sigma}\right)^2 + f_{22}\left(\frac{\sigma_2}{\sigma}\right)^2 + f_{66}\left(\frac{\sigma_{12}}{\sigma}\right)^2 + 2 f_{12}\left(\frac{\sigma_1\sigma_2}{\sigma^2}\right) \\
b &= f_1\left(\frac{\sigma_1}{\sigma}\right) + f_2\left(\frac{\sigma_2}{\sigma}\right) \\
f_{11} &= \frac{1}{XX'}, \quad f_{11} = \frac{1}{YY'}, \quad f_{66} = \frac{1}{S'^2} \\
f_1 &= \frac{1}{X} - \frac{1}{X'}, \quad f_2 = \frac{1}{Y} - \frac{1}{Y'} \\
f_{12} &\cong 0.5(f_{11}f_{22})^{1/2}
\end{aligned} \tag{29}$$

In Equations (23-29), E_1, E_2, ν_{12} and ν_{21} are the Young's modulus and Poisson's ratios in the principal coordinate system. Terms X and X' are the tensile and compressive strengths respectively in the longitudinal direction, Y and Y' are the tensile and compressive strengths in the transverse direction, while S' is the shear strength of the unidirectional layer.

CONCLUSION

The current work focused on calculating the stress concentration numerically using the developed finite element. In the first section of the present paper, the mathematical formulation of the developed element was presented in detail. Based on the classical plate theory, the finite element can be considered as a combination of two finite elements. The first one is a linear isoparametric membrane element. The second finite element is a high precision Hermitian element [40]. The first section is followed by an analytical review of some selected approaches, which can be used to determine the stress field in anisotropic plates under shear loading. In the last section of the present paper, the mathematical formulations of different failure theories were presented in detail to calculate the failure strength of laminated plates with and without cutouts.

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