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DAMAGE PREDICTION HYBRID PROCEDURE FOR FRP LAMINATES SUBJECTED TO RANDOM LOADS

In this paper, a hybrid procedure is formulated in order to predict the damage of a laminate composed of UD FRP laminae under random loading. This procedure is based on two pillars: a stiffness degradation model (SDM) combined with an energy approach taking into account the effect of load ratio in addition to a system of equations generated by SSDQM (space state differential quadrature method), which we solved with a novel technique. The outputs of SSDQM, previously used for free vibration behavior analysis of composite structures, are used with those of SDM to predict the damage failure of a composite laminate subjected to random loading. The obtained results correlate very well with the experimental ones and an extensive comparison with other models validate the accuracy and convergence characteristics of this hybrid procedure.

Keywords: **FRP laminate, random load, damage energy, lifetime prediction, stiffness degradation, load ratio**

INTRODUCTION

The life of a structure can come to a sudden end, or last longer, but only for a limited period. This latter case is usually accompanied by a reduction in yield, known as aging. Under a high load, a structure or component can deteriorate in one fell swoop, while it can actually withstand lower loads. On the other hand, the same structure or component can also be ruined under lower loads if they are applied over longer periods, either a constant amplitude (static) or variable amplitude (fatigue). These loadings sustained by mechanical structures are induced by external stresses (forces, thermal, accelerations, etc.). The phenomenon of the degradation of the properties of a material due to the application of loads that fluctuate over time is called fatigue and the resulting ruin is called fatigue failure.

Due to the complexity of the fatigue damage process in composite materials, predicting their fatigue life is of vital importance. Nonetheless, proper modeling of the damage evolution is the foundation for predicting the fatigue life of composite structures, which enables appropriate evaluation of the structure performance in its early cycles of life and prevents catastrophic failures. Some authors based their research on residual strength or stiffness. Yao and Himmel [1] predict the residual strength caused by fatigue damage in glass and carbon fiber reinforced plastics. To predict and investigate the effect of high-stress peaks on the fatigue life of carbon fiber reinforced plastics, Aghazadeh and Majidi [2] applied residual strength. Another stiffness-based model to predict lifetime developed in Ref. [3] is also considered quite a good model to predict the residual fatigue life of composites. However, the predictions employing these models noticeably diverge from the experimental values and mostly yield a high percent error in fatigue life prediction.

In recent decades another range of models, stiffness-based models [4-7], has been developed. The damage degree is quantified by measuring the stiffness of the material. Nevertheless, most of these models possess two major deficiencies, the first is a large number of parameters which require extensive experimental data to calculate them, while the second deficiency is their inability to accurately simulate the damage progress in its well-known three stages [7-9]. In addition to the aforementioned shortcomings, most of these models are used to validate a specific type of composite and cannot evaluate in a wide range of loading levels [1, 10, 11].

On the other hand, the vibration analysis of composite structures is also a large area of research encouraging researchers to ensure the usability, durability, and safety during the composite structure's lifetime. Many works have been conducted in this trend [12-22], and a number of models and methods have been developed. Among them the state space method combined with the differential quadrature method abbreviated as SSDQM stands out.

In this paper we aimed to resolve the discussed limitations of stiffness-based models by means of a hybrid damage prediction procedure. It consists in coupling a stiffness-based model with SSDQM, while relying both on an energy approach to predict damage rupture [23] and the well-known Palmgreen Miner rule [24]. In the first place, SSDQM is solved by a new proposed technique different from the ones found in the literature. A coupling algorithm is developed to survey the damage progress of the composite laminate and consequently predict its damage rupture. Numerical validation of the hybrid procedure demonstrates that most of the predicted lifetimes lead to quantitatively better estimations.

NOVEL TECHNIQUE FOR SOLVING SSDQM

In works [14, 15, 21], the authors combined the state space method (SSM) [17-20] with the differential quadrature method (DQM) [12-14] to establish an equation system (1) [17] and each one proceeds in his own way to solve it.

$$\frac{d\Delta}{dz} = M^{(k)}\,\Delta \tag{1}$$

where:

$$\Delta = \begin{bmatrix} Z & U & V & W & T_{xz} & T_{yz} \end{bmatrix}^T, Z = [Z_1 \;\; Z_2 \;\; \dots \;\; Z_N]^T;$$

$$M^{(k)} = \begin{bmatrix} 0 & M_1^{(k)} \\ M_2^{(k)} & 0 \end{bmatrix};$$

$$M_1^{(\mathrm{k})} = \begin{bmatrix} -\rho\omega^2 I & -g^{(1)} & \lambda_b I \\ -g^{(1)} & c_7 I & 0 \\ -\lambda_b I & 0 & \mathrm{c}_8 I \end{bmatrix};$$

$$M_2^{(k)} = \\ = \begin{bmatrix} c_9 I & c_1 g^{(1)} & -c_5\lambda_b I \\ c_1 g^{(1)} & \left(c_6\lambda_b^2 - \rho\omega^2\right)I - c_2 g^{(2)} & (c_3+c_6)\lambda_b g^{(1)} \\ c_5\lambda_b I & -(c_3+c_6)\lambda_b g^{(1)} & \left(c_4\lambda_b^2 - \rho\omega^2\right)I - c_6 g^{(2)} \end{bmatrix}$$

The components of matrix Δ are vectors defined as state variables vector *Z*. *N* is the discretization number, *k* represents the k^{th} ply of the laminate, *I* is the identity matrix and $g_{ij}^{(n)}$ are the weighting coefficients [16] dependent on Chebyshev-Gauss-Lobatto points x_i [17]:

$$x_i = \frac{a}{2}\left[1 - cos\frac{(i-1)\pi}{N-1}\right], \quad i = 1, 2, \dots, N,$$

Coefficients ci are defined and given in reference [17], which depends on the elastic material constants, ρ is the mass density and ω is a circular frequency, while λ_b is a constant parameter depending on arbitrary positive integer *n* and is expressed as the following: $\lambda_b = \frac{\pi n}{b}$.

For a specific problem, the boundary conditions at the edges (x = 0 and x = a) of the studied composite laminate must be taken into consideration so that a unique solution of equation (1) can be obtained. By applying boundary conditions, we add subscript '*q*' to equation (1) to indicate it:

$$\frac{d}{dz}\Delta_q = M_q^{(k)}\Delta_q \tag{2}$$

Explicit expressions of matrix $M_{1q}^{(k)}$ and $M_{2q}^{(k)}$ for each boundary condition case are given in Appendix B of Ref. [17].

Many methods are presented in the literature to solve system (2); Xu and Ding [22] used algebra rules and Cayley-Hamilton theorem to solve it. Direct use of the global transfer matrix is one of the methods found in literature [15-17] to solve this system. In this work, the global transfer matrix is also used to solve system (2) but in combination with the proposed coupling joint matrix and designated as J_C. The novel technique developed here to solve expression (2) consists in the following steps:

- First, the vector of the state variables for ply k is written as:

$$\Delta_i^{(k)} = \begin{Bmatrix} Z_i^{(k)} \\ U_i^{(k)} \\ V_i^{(k)} \\ W_i^{(k)} \\ T_{xz_i}^{(k)} \\ T_{yz_i}^{(k)} \end{Bmatrix} \tag{3}$$

where i takes '0' (inferior face of the ply) or '1' (superior face of the ply).

- Second, the following formula is supposed to assure the continuity condition between two adjacent plies: $J_C \cdot \begin{Bmatrix} \Delta_1^{(k)} \\ \Delta_0^{(k+1)} \end{Bmatrix} = 0$ where $J_C = [I \; -I]$ is called the coupling joint matrix which is composed of identity matrix *I* and its negative form (–*I*). It should be noted that identity matrix *I* has the same dimensions as the length of vector Δ.
- Third, the loading conditions at the superior interface and the inferior one are expressed respectively as: $J_{sup}.\Delta_1^m = f_{sup}$ and $J_{inf}.\Delta_0^1 = f_{inf}$. The inferior face does not submit any mechanical forces where vector force (stresses) f_{inf} is zero and consequently matrix J_{inf} is equal to zero. On the other hand, the laminate's superior face is submitted to bending loading where vector force f_{sup} and matrix J_{sup} are written as the following:

$$f_{sup} = \begin{Bmatrix} q_{sup} \\ 0 \\ 0 \end{Bmatrix} \tag{4}$$

$$J_{sup} = \begin{bmatrix} i_1 0 \; 0 \; 0 & 0 & 0 \\ 0 0 \; 0 \; 0 & i_5 & 0 \\ 0 0 \; 0 \; 0 & 0 & i_6 \end{bmatrix} \tag{5}$$

where i_1, i_5 and i_6 are the matrix identities having dimensions adequate to the state variables vector lengths Z, T_{xz} and T_{yz} respectively. We note also that the dimension of J_{inf} matrix is the same as matrix J_{sup}.

The gathering of all the above expressions of the joint coupling matrix lead to general formula (6):

$$J \cdot \Delta = f \tag{6}$$

with:

$$J = diag\,[J_{inf}\, J_{C_1} J_{C_2} \cdots J_{C_m} J_{sup}];$$

$$\Delta = [\left(\Delta_0^{(1)}\right)^T \begin{pmatrix} \Delta_1^{(1)} \\ \Delta_0^{(2)} \end{pmatrix}^T \cdots \begin{pmatrix} \Delta_1^{(m-1)} \\ \Delta_0^{(m)} \end{pmatrix}^T \left(\Delta_1^{(m)}\right)^T];$$

$f = [f_{inf}^T 0_1\, 0_2 \ldots 0_m f_{sup}^T]$ where 0_i is a zero vector of the i^{th} ply.

For any ply k of a composite laminate, the solution proposed to matrix system (2) is written as follows:

$$\begin{Bmatrix} \Delta_0^{(k)} \\ \Delta_1^{(k)} \end{Bmatrix} = M_q^{(k)} . \Delta_0^{(k)} \tag{7}$$

where:

$$M_q^{(k)} = \begin{bmatrix} I \\ T_k \end{bmatrix}; \qquad T_k = \exp\left(\frac{h_k}{h} . M_q^{(k)}\right).$$

Assembling all the plies of the laminate structure gives:

$$\Delta = M \cdot \Delta_0 \tag{8}$$

with

$$M = diag[M_1\, M_2 \ldots M_{m-1}\, M_m];$$

$$\Delta_0 = [(\Delta_0^{(1)})^T (\Delta_0^{(2)})^T \ldots (\Delta_0^{(m-1)})^T (\Delta_0^{(m)})^T]^T.$$

By substituting equation (8) into equation (6) system (9) is obtained:

$$J \cdot M \cdot \Delta_0 = f \tag{9}$$

Finally, the solution of system (9) gives all the state variable vectors in both the superior and inferior faces of each lamina.

LIFETIME ASSESSMENT PROCEDURE

Together with the outputs given by the solution of the system matrix developed in the previous section, the procedure that we aim to construct in this section is also based on both the stiffness degradation model (SDM) [25] and an algorithm used for an energy damage prediction model [23].

The stiffness degradation model category is one of the most popular manners to predict the damage of structures [4-7, 25, 26], which quantify the extent of damage by measuring the stiffness of the material. Formula (10) is used to construct the present procedure:

$$\frac{k_i}{k_0} = a_2 - a_1 ln\,\left(\frac{\frac{n_i}{N}}{1 - \frac{n_i}{N}}\right) \tag{10}$$

where a_1 and a_2 are the material parameters depending on ultimate static force F_r, stress ratio r_i and minimal force F_u for which failure is not reached. N is the critical lifetime for which the residual stiffness drops suddenly depending on the level of applied load.

The second pillar on which this procedure depends is the energy approach [23] used to determine two material parameters, $\emptyset$ and α, of formula (11), in addition to the lifetime at rupture N_{max}

$$N = \frac{N_{max}}{1 + e^{-\emptyset(\Psi + \alpha)}} \tag{11}$$

Based on these pillars, the algorithm developed for the procedure is scheduled as follows:

- Initially, Ergodic, Gaussian, stationary and random loading (designated as EGSR) is considered and thanks to the rainflow algorithm [23, 25] we obtain for each cycle 'i' mean value $F_{m,i}$ and amplitude $F_{a,i}$. These values obtained for elementary cycle 'i' are used as inputs for SSDQM to calculate the six components of displacements and stresses on each point of the discretized laminate. Then, maximum deflection $\delta_{max,i}$ occurring in the middle of the laminate is used to assess initial stiffness $k_{0,i}$ by means of expression (12); consequently, the minimal deformation energy of cycle 'i' is deduced via expression (13)

$$F_{a,i} = k_{0,i}\, \delta_{max,i} \tag{12}$$

$$\Psi_{min,i} = \frac{F_{a,i}^2}{2k_{0,i}} \tag{13}$$

- In the second step, SDM (10) is used to find final stiffness $k_{f,i}$ for each cycle 'i' which is used to determine the maximal deformation energy (14) of the considered cycle 'i'

$$\Psi_{max,i} = \frac{F_{a,i}^2}{2k_{f,i}} \tag{14}$$

- Thirdly, we calculate the loading parameter μ_i of each elementary cycle 'i' via expression (15). Therefore, by using the energy approach [23] material parameters $\emptyset_i$ and α_i as well as rupture lifetime N_i will be obtained

$$\mu_i = r_{,i} * \frac{F_{m,i}}{F_{max,i}} \tag{15}$$

- Finally, the well-known Palmgreen-Miner rule [24] is adopted to predict lifetime 'T' of the composite laminate examined as the following:

$$T = \frac{1}{\sum \frac{n_i}{N_i}} \tag{16}$$

To sum-up, the figure (Fig. 1) clearly shows the steps of the procedure developed above.

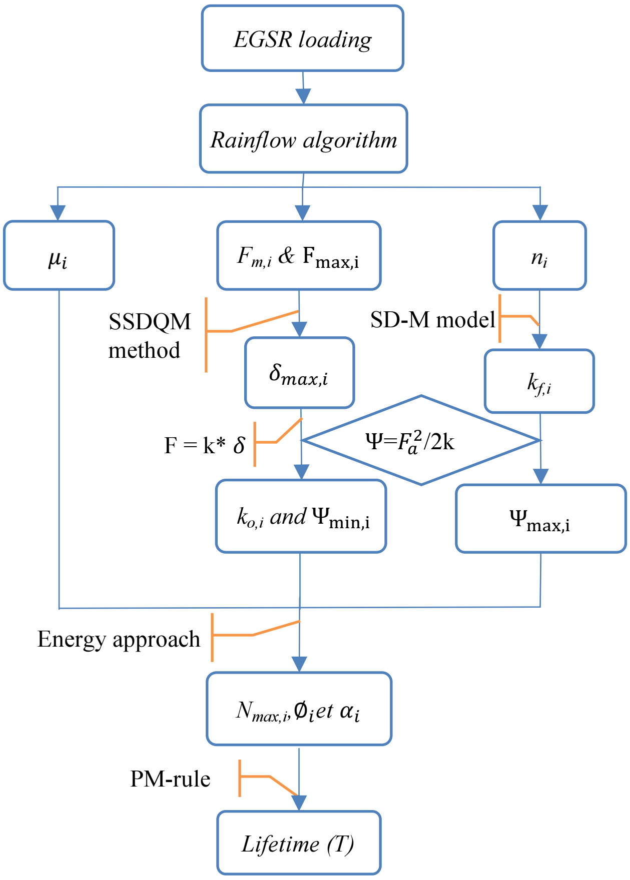

Fig. 1. Hybrid procedure for damage prediction of composite laminate

RESULTS AND DISCUSSION

The material used for the experimental study was a quasi-isotropic graphite/epoxy composite laminate $[45/0/90]_{3S}$ and the tests were performed on an MTS 810 servo-hydraulic machine [25, 27]. Two variable amplitude fatigue experiments were conducted where four specimens were tested for each experimental case defined by mean value F_m and standard deviation σ of the loading.

In order to validate the procedure developed in this work, two EGSR loadings were simulated identically to the experimental ones [25, 27]. In the first case, the mean value of loading was F_m = 2000 N and its standard deviation was σ = 500 N, while in the second case the mean value and the standard deviation were F_m = = 1500 N and σ = 350 N respectively. The experimental lifetimes are given in Table 1 together with our model predictions and some model results from the literature [23, 25, 27, 28].

The hybrid procedure predictions are quite identical to the energy model [23] results and are very close to both the statistical model [27] and the stiffness degradation model [25] ones. Niesloney's models [28] are too divergent in comparison with experimental tests [25] and all the models presented in Table 1.

On the other hand, we found that the difference between our procedure predictions and experimental lifetimes is very acceptable, especially for the second experiment which does not exceed 5 percent, whereas in the first experiment this percentage is much higher. Despite that, all the other models are also in the same order of magnitude as our hybrid model, but this difference can be explained by the fact that all these models used a linear cumulative damage rule to assess lifetimes which does not take into account the load sequence or interaction effects.

TABLE 1. Comparison between hybrid model predictions, experimental lifetimes, and some literature models

Lifetime (in cycles)	Loading 1: F_m = 2000 N σ = 500 N	Loading 2: F_m = 1500 N σ = 350 N
Experimental test [25]	4410	54517.5
Energy model [23]	6504.9	57909
Statistical model [27]	5900	58700
Stiffness degradation model [25]	5100	52300
Niesloney's first model [28]	1800	3000
Niesloney's second model [28]	3700	38000
Proposed hybrid procedure	6454.8	57883

CONCLUSIONS

The space state differential quadrature method is solved with a new technique and a series of stresses and deformations are obtained for each load cycle. SSDQM is usually used to analyze the free vibration behavior of different composite structures; in this work it was exploited in conjunction with one damage prediction model namely the stiffness degradation model (SDM) and an energy approach to predict the damage rupture of a composite laminate.

Satisfactory convergence of this hybrid procedure is verified by numerical comparison with other numerical models and also versus experimental tests. Hence, this procedure presents an ambition solution to monitor and predict damage evolution inside the laminate.

REFERENCES

[1] Yao W.X., Himmel N., A new cumulative fatigue model for fiber-reinforced plastics, Compos. Sci. Technol. 2000, 60, 59-64.

[2] Aghazadeh Mohandesi J., Majidi B., Fatigue damage accumulation in carbon/epoxy laminated composites, Mater. Des. 2009, 30, 1950-1956.

[3] Wu F., Yao W., A fatigue damage model of composite materials, Int. J. Fatigue 2010, 32 (1), 134-138.

[4] Jen Y.-M., Yang Y.-H., A study of two-stage cumulative fatigue behavior for CNT/epoxy composites, Proced. Eng. 2010, 2(1), 2111-2120.

[5] Toumi R.B., Renard J., Monin M., Nimdum P., Fatigue damage modelling of continuous E-glass fibre/epoxy composite, Proced. Eng. 2013, 66, 723-736.

[6] Zhang W., Zhou Z., Zhang B., Zhao S., A phenomenological fatigue life prediction model of glass fiber reinforced polymer composites, Mater. Des. 2015, 66, 77-81.

[7] Shiri S., Yazdani M., Pourgol-Mohammad M., Fatigue life prediction of polymeric composites based on the simultaneous degradation of stiffness and strength under two-stage loading, Modares Mech. Eng. 2015, 14(14), 137-142 (In Persian).

[8] Mao H., Mahadevan S., Fatigue damage modelling of composite materials, Compos. Struct. 2002, 58(4), 405-410.

[9] Kang K.-W., Lim D.-M., Kim J.-K., Probabilistic analysis for the fatigue life of carbon/epoxy laminates, Compos. Struct. 2008, 85(3), 258-264.

[10] Shiri S., Pourgol-Mohammad M., Yazdani M., Prediction of remaining fatigue cycles in composite materials under uncertainty, ASCE-ASME Journal of Risk and Uncertainty in Engineering Systems, Part B, Mechanical Engineering, 2015, http://dx.DOI. org/10.1115/1.4031037.

[11] Peng T., Liu Y., Saxena A., Goebel K., In-situ fatigue life prognosis for composite laminates based on stiffness degradation, Compos. Struct. 2015, 132, 155-165.

[12] Bellman R., Kashef BG., Casti J., Differential quadrature: a technique for the rapid solution of non-linear partial differential equations, J. Comput. Phys. 1972, 10, 40-52.

[13] Bert C.W., Jang S.K., Striz A.G., Nonlinear bending analysis of orthotropic rectangular plates by the method of differential quadrature, Comput. Mech. 1989, 5, 217-226.

[14] Bert C.W., Malik M., Transient analysis of gas-ubricated journal bearing systems by differential quadrature, ASME J. Tribol. 1997, 119, 91-99.

[15] Chen W.Q., Lv C.F., Bian Z.G., Elasticity solution for free vibration of laminated beams, Compos. Struct. 2003, 62, 75-82.

[16] Shu C., Richards B.E., Application of generalized differential quadrature to solve two-dimensional incompressible, Navier-Stokes equations, Int. J. Numer. Meth. Fluids 1992, 15, 791-798.

[17] Chen W.Q., Lue C.F., 3D free vibration analysis of cross-ply laminated plates with one pair of opposite edges simply supported, Comp. Struct. 2005, 69, 77-87.

[18] Bahar L.Y., Transfer matrix approach to layered systems, J. Eng. Mech. 1972, 98, 1159-1172.

[19] Chen W.Q., Cai J.B., Ye G.R., Exact solutions of cross-ply laminates with bonding imperfections, AIAA J. 2003, 41, 2244-2250.

[20] Chen W.Q., Lee K.Y., Three-dimensional exact analysis of angle-ply laminates in cylindrical bending with interfacial damage via state space method, Compos. Struct. 2004, 64, 275-283.

[21] Chen W.Q., Lüe C.F., 3D free vibration analysis of cross-ply laminated plates with one pair of opposite edges simply supported, Compos. Struct. 2005, 69, 77-87.

[22] Xu R., Ding H., Two-dimensional solutions for orthotropic materials by the state space method, Compos. Struct. 2007, 78, 325-336.

[23] Mohammed B., Mohamed A., Brahim O., Fatigue life prediction of composite laminate under random loading service: An energy approach, Materials Today: Proceedings, 2020, DOI: 10.1016/j.matpr.2020.05.091.

[24] Lee Y.-Li, Tjhung T., Chapter 3 – Rainflow Cycle Counting Techniques, Metal Fatigue Analysis Handbook 2012, 89-114.

[25] Bousfia M. et al., Fatigue life prediction for composite materials under Ergodic Gaussian and Stationary Random (EGSR) loads. Composites: Mechanics, Computations, Applications: An International Journal, 2017, DOI: 10.1615/CompMechComputApplIntJ.v8.i3.10.

[26] Shirazi A., Varvani-Farahani A., A stiffness degradation based fatigue damage model for FRP composites of (0/θ) laminate systems, Appl. Compos. Mater. 2010, 17(2), 137-150.

[27] Aboussaleh, M. Boukhili, R. Life prediction for composite laminates submitted to service loading spectra, Polym. Compos. 1998, 19, 241-245.

[28] Niesloney A., Böhom M., Mean stress effect correction using constant stress ratio S-N curves, Int. J. Fatigue 2013, DOI: 10.1016/j.ijfatigue.2013.02.019.

