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COMPOSITES THEORY AND PRACTICE

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MODELING AND OPTIMIZATION OF FIBER REINFORCED PRESSURE CHAMBER FOR SUBSEA APPLICATIONS

Since today's petrochemical industry is able to explore the seabed at the level of 3000 m, there is also a need to provide reliable, but cost effective enclosures for the respective technical equipment. This paper gives in-depth analysis of the design aspects for fiber reinforced pressure chambers, which are installed in the subsea environment. A chamber structure consisting of a metal liner and fiber reinforced composite is considered. Two theoretical models: for a generalized orthotropic material and for a laminated composite are recalled and used in the analytical study of the vessel. Finally, a structure optimization procedure is introduced and a verification example is provided.

Keywords: **pressure vessels, composite-reinforced metal cylinders, theory of orthotropic material, optimization**

MODELOWANIE I OPTYMALIZACJA ZBIORNIKA CIŚNIENIOWEGO WZMOCNIONEGO KOMPOZYTEM WŁÓKNISTYM DLA APLIKACJI PODWODNYCH

Dzisiejszy przemysł wydobycia ropy i gazu eksploruje dno morskie, sięgając trzykilometrowych głębin. Wysokie ciśnienie hydrostatyczne panujące na tym poziomie stawia znaczne wymagania wytrzymałościowe wobec zbiorników instalacji wydobywczej. W artykule przedstawiono zagadnienia związane z projektowaniem i optymalizacją zbiorników stalowych wzmocnionych kompozytem włóknistym. Przywołano teorię materiału ortotropowego oraz model laminatu kompozytowego, podano także analityczną metodę wymiarowania zbiornika wielowarstwowego. Zaprezentowano również metodę optymalizacji konstrukcji, która pozwoliła uzyskać znaczną sztywność zbiornika ciśnieniowego przy zachowaniu jego małej masy.

Słowa kluczowe: **zbiornik ciśnieniowy, cylinder stalowy wzmocniony kompozytem, materiał ortotropowy, optymalizacja**

INTRODUCTION

Since today's petrochemical engineering has reached the seabed at the level of 3000 meters, technological chambers installed on the seabed must be treated as high pressure vessels. The load value of 300 bars applied to the walls of containers holding technical and process equipment sets very high mechanical requirements for tank structures. However in addition, the transportation and installation restrictions require the vessels to be very lightweight. Thus, designing subsea containers for the oil and gas industry is not a trivial task since it involves an intensive optimization procedure aimed at meeting various, and often contradicting constraints. For this reason, hybrid constructions, made of different materials, are seriously considered in this application. Besides pure mechanical requirements, the usage of a metallic material allows for electro-magnetic shielding, while the composite shell provides good protection against the salty environment. This paper investigates the mechanical behavior of a metal-composite structure being exposed to high pressure load, and proposes an optimization procedure aiming to increase the strength-per-weight ratio. The proposed approach determines the optimal layers thicknesses and winding angles for the composite plies.

The theory of pressurized vessels has been studied extensively over several decades, therefore the literature is quite rich in this context. Steel tubes exposed to a pressure load have been investigated for example by Hill [1], Mendeleson [2], Chakrabarty [3], Fryer and Harvey [4]. There is also quite an extensive collection of standards, manuals and practical-oriented guides [5-7] being used by different businesses. The application of composite materials for pressurized vessels is similarly quite well deliberated. The literature includes the methods proposed by Durban [8], Zhao [9], Perry [10], and many others. Some specific cases of balanced composites have been studied by Niezgoda and Klasztorny [11], followed by Lewiński and Wilczyński [12], who derived stress-strain relations and material constants for diagonal laminates. The technical aspects for manufacturing wound composite structures for pressurized vessels have been raised in several papers presented by Błażejewski, for example [13, 14]. Nevertheless, most of the studies managed so far have focused on pressurized steel tubes or fiber-reinforced composite cylinders, while very limited studies have been published on multi-layer, metal-composite structures. Some ideas have been provided in [15, 16].

The work described in this paper considers a hybrid type of metal-composite structure, aiming to form a high-pressure vessel. A composite overwrapped steel cylinder exposed to hydrostatic pressure has been designed and evaluated analytically, as well as numerically by the Finite Element Method. The analyzed structure exhibited superior mechanical performance and strength-per-weight ratio, by optimal assignment of the winding angle and layer thickness.

The paper is formatted as follows: First, there are presented the analytical calculation approach for the metal-composite structure, giving insight into the material models and the constitutive relations. Next, there covers the optimization procedure provides the numerical example. The concluding remarks are gathered in the last part of the work.

ANALYTICAL CALCULATION MODEL

Systematic analysis of the problem requires determination of the equilibrium relations, as well as constitutive equations for the fiber reinforced metal cylinder, assuming some equivalent mechanical properties of the composite. The material modules of the laminated structure will be derived based on the generalized theory for orthotropic materials first, and simplified later into the classical lamination model.

Orthotropic material model

The generalized Hooke's Law for an orthotropic material may be found in many textbooks [17-19], and its compliance form usually can be given as:

$$\begin{Bmatrix} \varepsilon_{LL} \\ \varepsilon_{TT} \\ \varepsilon_{ZZ} \\ \gamma_{TZ} \\ \gamma_{ZL} \\ \gamma_{LT} \end{Bmatrix} = \begin{vmatrix} \frac{1}{E_L} & \frac{-\upsilon_{TL}}{E_T} & \frac{-\upsilon_{ZL}}{E_Z} & 0 & 0 & 0 \\ \frac{-\upsilon_{LT}}{E_L} & \frac{1}{E_T} & \frac{-\upsilon_{ZT}}{E_Z} & 0 & 0 & 0 \\ \frac{-\upsilon_{LZ}}{E_L} & \frac{-\upsilon_{TZ}}{E_T} & \frac{1}{E_Z} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{TZ}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{ZL}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{LT}} \end{vmatrix} \begin{Bmatrix} \sigma_{LL} \\ \sigma_{TT} \\ \sigma_{ZZ} \\ \tau_{TZ} \\ \tau_{ZL} \\ \tau_{LT} \end{Bmatrix} \quad (1)$$

where: E_L, E_T and E_Z represent Young's Modulus for the ply in Longitudinal (L), and Transverse (T and Z) directions, correspondingly. G and ν with their respective indices are the Kirchhoff modulus and Poisson ratio.

For flat composite layouts of the same fiber orientation, both transverse directions are equivalent (transversely isotropic material), therefore also the respective material properties are equivalent: $E_T = E_Z$, $\nu_{LT} = \nu_{LZ}$, $\nu_{TZ} = \nu_{ZT}$, $G_{LT} = G_{LZ}$, $G_{TZ} = E_T / 2(1+\nu_{TZ})$.

If a composite material layout is rotated by angle α about the Z axis - transverse to the fiber direction, its local coordinate system (LTZ) is not aligned any more with the global one (XYZ), Figure 1.

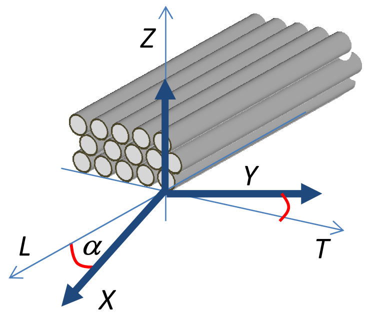

Fig. 1. Rotation about orthotropic transverse axis

Rys. 1. Obrót wokół osi poprzecznej włókien

In this case, the compliance matrix appearing in Hooke's Law must be treated by the transformation matrix of direction cosines, $[T]_{6X6}$, and it can be expressed now as:

$$\begin{Bmatrix} \varepsilon_{XX} \\ \varepsilon_{YY} \\ \varepsilon_{ZZ} \\ \gamma_{YZ} \\ \gamma_{ZX} \\ \gamma_{XY} \end{Bmatrix} = \begin{vmatrix} \frac{1}{E_X} & \frac{-\upsilon_{YX}}{E_Y} & \frac{-\upsilon_{ZX}}{E_Z} & 0 & 0 & \frac{\eta_{XY}}{G_{XY}} \\ \frac{-\upsilon_{XY}}{E_X} & \frac{1}{E_Y} & \frac{-\upsilon_{ZY}}{E_Z} & 0 & 0 & \frac{\mu_{XY}}{G_{XY}} \\ \frac{-\upsilon_{XZ}}{E_X} & \frac{-\upsilon_{YZ}}{E_Y} & \frac{1}{E_Z} & 0 & 0 & \frac{\varsigma_{XY}}{G_{XY}} \\ 0 & 0 & 0 & \frac{1}{G_{YZ}} & \frac{\xi_{XZ}}{G_{XZ}} & 0 \\ 0 & 0 & 0 & \frac{\xi_{YZ}}{G_{YZ}} & \frac{1}{G_{XZ}} & 0 \\ \frac{\eta_X}{E_X} & \frac{\mu_Y}{E_Y} & \frac{\varsigma_Z}{E_Z} & 0 & 0 & \frac{1}{G_{XY}} \end{vmatrix} \begin{Bmatrix} \sigma_{XX} \\ \sigma_{YY} \\ \sigma_{ZZ} \\ \tau_{YZ} \\ \tau_{ZX} \\ \tau_{XY} \end{Bmatrix} \quad (2)$$

The transformation matrix is not shown here for the sake of clarity, but detailed explanation of this topic can be found, for example, in [17].

The reader should note however, non-zero coefficients (η, μ, ζ) which appeared between the volumetric and shear factors. Their presence indicates that rotation of the orthotropic transverse axis causes quite complex interactions between the strains and stresses. For example, the axial load applied to the coil spring will produce axial extension of the spring and a change in its diameter (which is naturally expected), but also the rotation of the spring turns.

In the case of thin composite layouts, eq. (2) can be limited to 2D space only, and simplified further to:

$$\begin{Bmatrix} \varepsilon_{XX} \\ \varepsilon_{YY} \\ \gamma_{XY}/2 \end{Bmatrix} = \begin{vmatrix} 1/E_X & -\nu_{YX}/E_Y & \eta_{XY}/G_{XY} \\ -\nu_{XY}/E_X & 1/E_Y & \mu_{XY}/G_{XY} \\ \eta_X/2E_X & \mu_Y/2E_Y & 1/2G_{XY} \end{vmatrix} \begin{Bmatrix} \sigma_{XX} \\ \sigma_{YY} \\ \tau_{XY} \end{Bmatrix} \quad (3)$$

where:

$$E_X = \frac{1}{\dfrac{c^4}{E_L} + \dfrac{s^4}{E_T} + c^2 s^2 \left(\dfrac{1}{G_{LT}} - \dfrac{2\upsilon_{TL}}{E_T} \right)}$$

$$E_Y = \frac{1}{\dfrac{s^4}{E_L} + \dfrac{c^4}{E_T} + c^2 s^2 \left(\dfrac{1}{G_{LT}} - \dfrac{2\upsilon_{TL}}{E_T} \right)}$$

$$G_{XY} = \frac{1}{4c^2 s^2 \left(\dfrac{1}{E_T} + \dfrac{1}{E_L} + 2\dfrac{\nu_{TL}}{E_T} \right) + \dfrac{(c^2 - s^2)^2}{G_{LT}}}$$

$$\upsilon_{YX} = E_Y \left(\frac{\upsilon_{TL}}{E_T}(c^4 + s^4) - c^2 s^2 \left(\frac{1}{E_T} + \frac{1}{E_L} - \frac{1}{G_{LT}} \right) \right) \qquad \upsilon_{XY} = \upsilon_{YX} \frac{E_X}{E_Y}$$

$$\eta_{XY} = G_{XY} \left[2cs \left\{ \frac{s^2}{E_T} - \frac{c^2}{E_L} - (c^2 - s^2) \left(\frac{\upsilon_{TL}}{E_T} - \frac{1}{2G_{LT}} \right) \right\} \right] \qquad \eta_X = \eta_{XY} \frac{E_X}{G_{XY}}$$

$$\mu_{XY} = G_{XY} \left[2cs \left\{ \frac{c^2}{E_T} - \frac{s^2}{E_L} + (c^2 - s^2) \left(\frac{\upsilon_{TL}}{E_T} - \frac{1}{2G_{LT}} \right) \right\} \right] \qquad \mu_Y = \mu_{XY} \frac{E_Y}{G_{XY}}$$

$s = \sin\alpha$, $c = \cos\alpha$ α = winding angle

The derivation of the material properties in the global co-ordinate system (E_X, E_Y, G_{XY}, …) requires tedious mathematical operations involving rotation matrices [T], thus, again, only the final results are shown above.

In contrast, the necessary material properties of a single ply in a fiber co-ordinate system (E_L, E_T, G_{LT}, …) are normally provided by the composite manufacturer. However, if not given by the producer, they may be easily derived by the concept of the commonly used Rule of Mixtures:

$$E_L = fE_f + (1-f)E_m \qquad E_T = \frac{E_m E_f}{fE_m + (1-f)E_f}$$
$$\nu_{LT} = f\nu_f + (1-f)\nu_m \qquad \nu_{TL} = \frac{E_T}{E_L}\nu_{LT} \tag{4}$$
$$G_{LT} = \frac{G_m G_f}{fG_m + (1-f)G_f}$$

where: f is the fiber volume fraction (typically about 0.6), and indices "m" and "f" refer to the material properties of the matrix and fiber, respectively.

In order to homogenize several layers of different materials or winding angles, it is necessary to provide stiffness matrixes [K_i] for all the plies first, next sum them up with respect to the layer thickness to form global stiffness matrix [K_G], and finally compute the equivalent material properties out of global compliance matrix $[K_G]^{-1}$.

The stiffness matrix, [K_i], for an individual layer can be calculated by inverting the local compliance matrix given by (3)

$$[K_i] = \frac{1}{m} \begin{vmatrix} E_X\left(\dfrac{E_Y}{E_X}\mu_{XY}\mu_Y - 1\right) & -E_X\left(\dfrac{E_Y}{E_X}\eta_{XY}\mu_Y + \nu_{XY}\right) & 2E_X(\mu_{XY}\nu_{YX} + \eta_{XY}) \\ -E_Y(\mu_{XY}\eta_X + \nu_{XY}) & E_Y(\eta_{XY}\eta_X - 1) & 2E_Y(\eta_{XY}\nu_{XY} + \mu_{XY}) \\ G_{XY}\left(\dfrac{E_Y}{E_X}\mu_Y\nu_{XY} + \eta_X\right) & G_{XY}\left(\eta_X\nu_{YX} + \dfrac{E_Y}{E_X}\mu_Y\right) & 2G_{XY}(\nu_{XY}\nu_{YX} - 1) \end{vmatrix}$$

$$m = (\nu_{XY} + \mu_{XY}\eta_X)\nu_{YX} + \left(\frac{E_Y}{E_X}\mu_Y\nu_{XY} + \eta_X\right)\eta_{XY} + \frac{E_Y}{E_X}\mu_{XY}\mu_Y - 1 \tag{5}$$

Since the composite consists of several plies, global stiffness matrix [K_G] for the whole structure is derived with respect to the thicknesses of individual plies:

$$[K_G] = \frac{1}{t}\sum_i^N t_i [K_i] \tag{6}$$

where: t_i - thickness of i-th ply, t - total thickness of all, N - plies.

Finally, in order to determine the equivalent mechanical properties of the multi-layer composite, it is necessary to calculate the inverse of the global stiffness matrix, $[K_G]^{-1}$ first, and perform simple operations on its components:

$$[K_G]^{-1} = \begin{vmatrix} k_G^{11} & k_G^{12} & k_G^{13} \\ k_G^{21} & k_G^{22} & k_G^{23} \\ k_G^{31} & k_G^{32} & k_G^{33} \end{vmatrix} \rightarrow \begin{cases} \overline{E}_X = \dfrac{1}{k_G^{11}} & \overline{\nu}_{YX} = -\dfrac{k_G^{12}}{k_G^{22}} & \overline{\eta}_{XY} = \dfrac{k_G^{13}}{2k_G^{33}} \\ \overline{\nu}_{XY} = -\dfrac{k_G^{21}}{k_G^{11}} & \overline{E}_Y = \dfrac{1}{k_G^{22}} & \overline{\mu}_{XY} = \dfrac{k_G^{23}}{2k_G^{33}} \\ \overline{\eta}_X = \dfrac{k_G^{31}}{2k_G^{11}} & \overline{\mu}_Y = \dfrac{k_G^{32}}{2k_G^{22}} & \overline{G}_{XY} = \dfrac{1}{2k_G^{33}} \end{cases} \tag{7}$$

It is worth noting that since the composite structure is not symmetrical, coupling terms (η, μ) contribute to the material properties. This complex interaction will not appear however, if the composite layup is made by the number of alternate layers, which are balanced symmetrically: $+\alpha/-\alpha$. In this case, the coupling factors will vanish, and a much simpler model of classical lamination may be considered.

Classical Lamination model

One may show [20-22], that the mechanical properties of a single ply, (4), form the local stiffness matrix, [k_i], (in the local coordinate system LTZ, Figure 1):

$$[k_i] = \begin{vmatrix} \dfrac{E_L}{1-\nu_{LT}\nu_{TL}} & \dfrac{\nu_{LT}E_T}{1-\nu_{LT}\nu_{TL}} & 0 \\ \dfrac{\nu_{TL}E_L}{1-\nu_{LT}\nu_{TL}} & \dfrac{E_T}{1-\nu_{LT}\nu_{TL}} & 0 \\ 0 & 0 & 2G_{LT} \end{vmatrix} \tag{8}$$

Because the longitudinal direction of a single i-th ply may be arbitrarily oriented in space (rotated about the helical angle, α), therefore it is also necessary to transform the local stiffness matrix into the global co-ordinate system, [K_i]:

$$[K_i] = T[k_i]T^{-1} \tag{9}$$

where [K_i] is the stiffness matrix of i-th ply in the global coordinate system, and T - is the rotation matrix:

$$T = \begin{bmatrix} c^2 & s^2 & 2sc \\ s^2 & c^2 & -2sc \\ -sc & sc & c^2 - s^2 \end{bmatrix} \tag{10}$$

where: s - $\sin\alpha$, c - $\cos\alpha$, α - winding angle.

Next, all stiffness matrices $[K_i]$ of the individual plies must be aggregated into global stiffness matrix $[K_G]$, as given by (6), and, after inverting, the equivalent properties of the composite may be derived finally as:

$$[K_G]^{-1} = \begin{vmatrix} k_G^{11} & k_G^{12} & 0 \\ k_G^{21} & k_G^{22} & 0 \\ 0 & 0 & k_G^{33} \end{vmatrix} \rightarrow \begin{cases} \overline{E}_X = \dfrac{1}{k_G^{11}} & \overline{\nu}_{YX} = -\dfrac{k_G^{12}}{k_G^{22}} & . \\ \overline{\nu}_{XY} = -\dfrac{k_G^{21}}{k_G^{11}} & \overline{E}_Y = \dfrac{1}{k_G^{22}} & . \\ . & . & \overline{G}_{XY} = \dfrac{1}{2k_G^{33}} \end{cases} \tag{11}$$

One may find that the classical lamination model is much simpler than generalized orthotropic theory, however, both ways produce the same equivalent material properties for balanced composite structures.

It must be also noted that (local) stresses in a specific *i*-th layer of the composite can be determined by:

$$\begin{Bmatrix} \sigma_L \\ \sigma_T \\ \gamma_{LT} \end{Bmatrix}_i = [k_i][T]^{-1}[K_G]^{-1} \begin{Bmatrix} \sigma_X \\ \sigma_Y \\ \gamma_{XY} \end{Bmatrix} \tag{12}$$

Naturally, the calculated stresses for a single, *i*-th ply (σ_L, σ_T) should be smaller than the allowed strength limits for the fiber and matrix, respectively.

Equilibrium and Constitutive relations

In order to analytically calculate the stress values in the metal tube and the composite structure, the following assumptions have been applied:

- the object under study is constructed out of two, co-axial cylindrical parts, made of different materials, as shown schematically in Figure 2. The inner part, marked as ”1”, represents the metal liner (having the thickness e_1); and the outer part, marked as ”2”, symbolizing the composite shell (having the total thickness e_2),
- the composite shell consists of several plies oriented at different helical angles, and embodied in the epoxy resin matrix,
- the orientation and number of fiber layers are unrestricted, but should form a balanced structure ($+\alpha$ /$-\alpha$), and the entire thickness of all the layers constitutes the outer tube ($\Sigma t_i = e_2$),
- the steel cylinder (inner tube) and the composite cylinder (outer tube) assume linear-elastic material models,
- both tubes: steel and composite are perfectly bounded,
- because the total thickness of the structure is low with respect to the cylinder radius, r/e >10, the theory of thin-walled shells can be applied.

To determine the stress and strain fields in both the steel liner and the composite shell, one should apply equilibrium and constitutive equations. On the basis of the first relation, Figure 3, it is possible to state:

$$\begin{cases} 1 \times 2(e_1\sigma_{1X} + e_2\sigma_{2X}) = 1 \times 2rp_0 \\ 2\pi r(e_1\sigma_{1Y} + e_2\sigma_{2Y}) = \pi r^2 p_0 \end{cases} \tag{13}$$

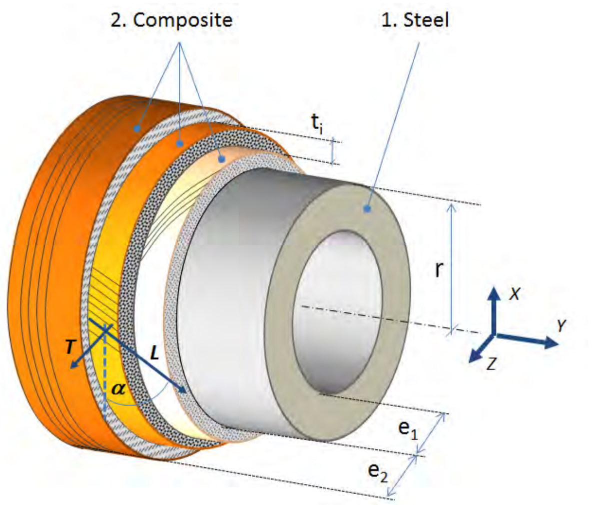

Fig. 2. Geometrical model of cylinder under study
Rys. 2. Model geometryczny zbiornika wielowarstwowego

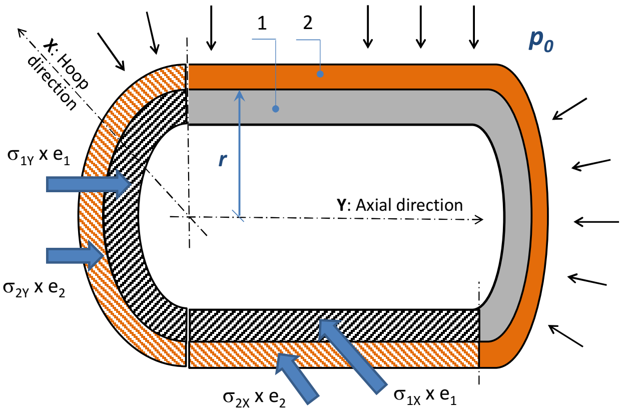

Fig. 3. Equilibrium relations for axial and hoop directions
Rys. 3. Warunki równowagi dla zbiornika ciśnieniowego

Because of the different materials used, constitutive equations should be written for the steel tube and the composite tube independently, with respective indices “1” and “2”:

$$\begin{Bmatrix} \varepsilon_{1X} \\ \varepsilon_{1Y} \\ \gamma_{1XY}/2 \end{Bmatrix} = \begin{vmatrix} 1/E_1 & -\nu_1/E_1 & 0 \\ -\nu_1/E_1 & 1/E_1 & 0 \\ 0 & 0 & 1/2G_1 \end{vmatrix} \begin{Bmatrix} \sigma_{1X} \\ \sigma_{1Y} \\ \tau_{1XY} \end{Bmatrix} \tag{14}$$

$$\begin{Bmatrix} \varepsilon_{2X} \\ \varepsilon_{2Y} \\ \gamma_{2XY}/2 \end{Bmatrix} = \begin{vmatrix} 1/\overline{E}_X & -\overline{\nu}_{YX}/\overline{E}_Y & 0 \\ -\overline{\nu}_{XY}/\overline{E}_X & 1/\overline{E}_Y & 0 \\ 0 & 0 & 1/2\overline{G}_{XY} \end{vmatrix} \begin{Bmatrix} \sigma_{2X} \\ \sigma_{2Y} \\ \tau_{2XY} \end{Bmatrix} \tag{15}$$

Please note, that the “dash” sign in (15) points out that there are equivalent properties used for the composite, compare (11).

Fulfilling the assumption that the steel and the composite tubes are bounded, one should also state that their strains in respective directions are equal each other:

$$\varepsilon_{1X} = \varepsilon_{2X} \quad \text{and} \quad \varepsilon_{1Y} = \varepsilon_{2Y} \tag{16}$$

The remaining condition on the equality of shear strains, γ, can be omitted since it is automatically satisfied in the case of the composite having plies oriented symmetrically ($+\alpha/-\alpha$).

Relations (13-16) constitute a system of four equations with the same number of unknowns (stress vector):

$$\begin{bmatrix} e_1 & 0 & e_2 & 0 \\ 0 & e_1 & 0 & e_2 \\ \frac{1}{E_1} & -\frac{\nu_1}{E_1} & -\frac{1}{\overline{E}_X} & \frac{\overline{\nu}_{YX}}{\overline{E}_Y} \\ -\frac{\nu_1}{E_1} & \frac{1}{E_1} & \frac{\overline{\nu}_{XY}}{\overline{E}_X} & -\frac{1}{\overline{E}_Y} \end{bmatrix} \begin{Bmatrix} \sigma_{1X} \\ \sigma_{1Y} \\ \sigma_{2X} \\ \sigma_{2Y} \end{Bmatrix} = \begin{Bmatrix} p_0 r \\ \frac{p_0 r}{2} \\ 0 \\ 0 \end{Bmatrix} \tag{17}$$

Please notice that (17) can also be applied to the design of all types of hydraulic actuators and hydraulic cylinders, if the "$p_0r/2$" term in the load vector is replaced by "$p_0r/2 - F/2\pi r$" (where F is an additional axial force). The unknowns in (17) may be easily derived numerically, but mathematical purists may also provide their complex symbolic solution:

$$\begin{Bmatrix} \sigma_{1X} \\ \sigma_{1Y} \\ \sigma_{2X} \\ \sigma_{2Y} \end{Bmatrix} = \frac{p_0 r}{2k} \begin{Bmatrix} E_1\left[2e_1E_1\left(\overline{\nu}_{XY}\overline{\nu}_{YX}-1\right)+e_2\left(\overline{E}_X\overline{\nu}_{YX}+2\overline{E}_Y\nu_1\overline{\nu}_{XY}-\overline{E}_X\nu_1-2\overline{E}_Y\right)\right] \\ E_1\left[e_1E_1\left(\overline{\nu}_{XY}\overline{\nu}_{YX}-1\right)+e_2\left(\overline{E}_X\nu_1\overline{\nu}_{YX}+2\overline{E}_Y\overline{\nu}_{XY}-\overline{E}_X-2\overline{E}_Y\nu_1\right)\right] \\ \overline{E}_X\left[e_1E_1\left(2\overline{\nu}_{XY}\overline{\nu}_{YX}-\overline{\nu}_{YX}+\nu_1-2\right)+2e_2\overline{E}_Y\left(\nu_1^2-1\right)\right] \\ \overline{E}_Y\left[e_1E_1\left(\nu_1\overline{\nu}_{XY}-2\overline{\nu}_{XY}+2\nu_1-1\right)+e_2\overline{E}_X\left(\nu_1^2-1\right)\right] \end{Bmatrix}$$

where:

$$k = e_1^2E_1^2\left(\overline{\nu}_{XY}\overline{\nu}_{YX}-1\right)+e_1e_2E_1\left[\overline{E}_X\left(\nu_1\overline{\nu}_{YX}-1\right)+\overline{E}_Y\left(\nu_1\overline{\nu}_{XY}-1\right)\right]-e_2^2\overline{E}_X\overline{E}_Y \tag{18}$$

Eq. (18) gives directly the values of the hoop and axial stresses in both the metal tube and the composite shell. However, the stresses in the composite must be treated as equivalent ones since the material properties were homogenized. In order to calculate the stresses in specific plies of the composite, the respective results of (18), namely: σ_{2X}, σ_{2Y} must be introduced into (12).

OPTIMIZATION PROCEDURE

As stated in the Introduction, the aim of this study was to provide the design of a subsea vessel, which offers superior power-per-weight properties. For this reason, the objective of the optimization procedure was to minimize the goal function, GF, given as follows:

$$\min(GF) \rightarrow \min\left(e_1 \cdot \rho_{Steel} + \sum_{i=1}^{N} t_i \cdot \rho_i\right) \tag{19}$$

where: e_1 - is the thickness of the steel tube, t_i - is the thickness of the i-th ply, ρ - density of the layer material, and N - max number of balanced plies ($+/-\alpha$) in the composite structure. Naturally, the density value of a single ply should be derived with respect to the volume fraction of fibers (f) and the resin matrix ($1-f$).

One may note that goal function (19) is proportional to the weight of the vessel. However, if the density is replaced by the cost of the material unit [$/kg], the objective will minimize the overall cost of the analyzed chamber.

The design vector, which groups the design variables to be modified during the optimization, covers:

$$V = [e_1, t_i, \alpha_i] \quad \text{for } i = 1..N \tag{20}$$

In other words, it is assumed that the thickness of the steel tube, as well as the thicknesses and orientation angles for all the plies constituting the composite structure, are the optimization variables to play with.

The constraints of the optimization procedure refer to the material strength limits, and geometrical bounds for the design variables:

$$C = \begin{bmatrix} \sigma_{Li} \le \sigma_{LiMAX} \\ \sigma_{Ti} \le \sigma_{TiMAX} \\ \sigma_{SteelMises} \le \sigma_{SteelMisesMAX} \\ 0.5 \le t_i \le 10 \\ 0 \le \alpha_i \le 90 \\ 3 \le e_1 \le 10 \end{bmatrix} \quad \text{for } i = 1..N \tag{21}$$

Please note, that the failure criteria proposed in (21) are very simple, but formally, any other model for the evaluation of a composite fracture, like the Hill criterion or Tsai-Wu theory, for example, can also be applied here.

Having the optimization problem fully defined by (19)-(21), one may solve it with a dedicated piece of software, or just using a "brutal force" method. The author applied in practice the SOLVER add-in of MS/Excel, since this module is easily accessible. However, any other tool or method can be equally used.

To summarize, the complete calculation procedure looks as follows:

- first, the input data are assumed: testing pressure (p_0), radius of the vessel (R), number of different plies (N), type of fibers and the steel grade. The respective constraints are set (21): material strength limits (σ_{MAX}), as well as manufacturing restrictions (e.g. minimal thickness of composite ply and metal tube),
- the design vector (20) is initiated with some preliminary values (e.g. e_1 = 5 mm; t_1 = 2 mm; $\alpha_1 = \pm 15°$, …),
- material properties for individual plies in their longitudinal and transverse directions are calculated (4), and the corresponding local stiffness matrixes are derived (8),

- the properties of individual plies are calculated in GCS (9)-(10), and assembled into the global stiffness matrix (6),
- the global stiffness matrix is inversed, and equivalent material properties of the complete composite are calculated (11),
- stress components in the hoop and axial directions of GCS are derived (18),
- the stress intensity (Mises) is calculated for the steel tube, while longitudinal and transverse stress components for the composite plies are found based on (12),
- if all stress constrains (21) are satisfied, the goal function (19) is calculated, and a new, more aggressive design vector (20) is proposed; if the stress constraints are not met - then a bit safer design vector is assumed,
- the calculation loop is repeated until all the constraints are fulfilled, and the best solution (the minimum value of the goal function) is found.

NUMERICAL EXAMPLE

The numerical calculations were performed for the vessel having radius R = 100 mm, while the testing pressure was assumed to be p_0 = –30 MPa (the minus sign /-/ of hydrostatic load follows the convention of the external pressure load being negative, and internal one positive). Typical steel material was used (E = 205 GPa, ν = 0.33), whereas two different types of fibers were considered: carbon (E = 240 GPa, ν = 0.20), and basalt (E = 88 GPa, ν = 0.20), being embedded in the ordinary epoxy resin matrix (E = 3 GPa, ν = 0.40). The material strength limits were set to 370 MPa (Mises) in the case of the steel tube, 1800 MPa for fibers (in longitudinal direction), and 60 MPa for the resin (transverse direction). The optimization problem, as described above (19)-(21), was introduced into the standard MS/Excel environment, Figure 4.

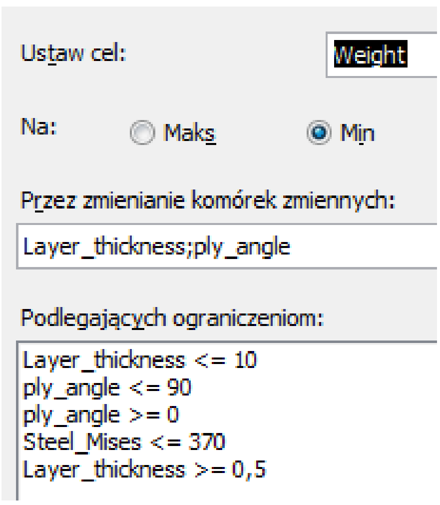

Fig. 4. Optimization settings of SOLVER module

Rys. 4. Ustawienia zadania optymalizacyjnego

The evolutionary algorithm, which was used in this case, determined the optimal design to be: 4.8 mm steel /4.2 mm 0˚ carbon fiber/1 mm ±20˚ basalt fiber. The calculated stresses reached the assumed limit (370 MPa) in the case of the steel tube, while 224.7 and 80.7 MPa (compression) were achieved in the fiber direction of the carbon and basalt layers, respectively. To validate these values, the same problem was modeled using ABAQUS, a general purpose FEM package [23]. The results of this study were in excellent agreement with the analytical calculation (below 1% difference), which basically should not be surprising, since most FEM software packages apply the same classical lamination model, as described above.

CONCLUSIONS

This paper provided the structured approach for the optimal design of composite reinforced steel vessels used in subsea applications. The background information about the classical lamination model was given first, and next applied in the optimization procedure. The presented methodology offers the same quality of results as FEM calculations, but is much faster and thus allows for immediate optimization. One should also note that it can be equally applied to the design of all types of hydraulic actuators and hydraulic cylinders if some minor modifications are introduced. Further study will focus the thermal aspects of the vessel construction, since the external surface of the chamber is cooled by seawater (4˚C), while the internal one is heated by the industrial equipment mounted inside (70˚C).

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