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INVESTIGATION OF DYNAMIC BEHAVIOUR OF LAMINATED COMPOSITE PLATES UNDER CYCLIC LOADING

Polymeric laminated composites show great potential in many engineering applications, especially in the aircraft industry, owing to their specific strength properties. Many machine components made of polymeric laminates are subjected to intensive vibrations. According to the viscoelastic nature of such composites, some specific effects e.g. energy dissipation could be observed during cyclic vibrations. Therefore, it is necessary to understand this behaviour and to develop appropriate methods and models for diagnostics and monitoring purposes. In this work the authors present the results of an experimental investigation into the vibration response of cyclically loaded glass-fiber reinforced polymeric (GFRP) rectangular plates. Initially, the frequency response functions (FRF) for the investigated specimens of various lengths were obtained during laboratory experiments. The natural frequencies of the specimens were determined based on their FRF. Then, the specimens were loaded on the first three natural frequencies to obtain characteristics showing the phenomenon of energy dissipation. Dynamic testing was carried out using a laser vibrometer and a piezoelectric force sensor. The evolution of the dynamic moduli was investigated based upon the measurement results, allowing estimation of the empirical model of material energy loss, necessary in building and testing the analytical model of the self-heating properties of the specimen material. The influence of an excitation frequency and the length of the specimens on their dynamic behaviour was additionally studied. The results of the conducted research could be successfully applied in diagnostics and structure health monitoring (SHM) applications and could be used to develop fatigue and fracture models of viscoelastic GFRP laminated composites.

Keywords: polymeric laminated composites, dynamic response, hysteresis, dynamic moduli

BADANIE ZACHOWANIA DYNAMICZNEGO LAMINOWANYCH PŁYT KOMPOZYTOWYCH PODCZAS OBCIĄŻEŃ CYKLICZNYCH

Laminowane kompozyty polimerowe z uwagi na wyjątkowe własności wytrzymałościowe mają duży potencjał do zastosowań w wielu aplikacjach inżynierskich, zwłaszcza w przemyśle lotniczym, motoryzacyjnym czy też maszynowym. Wiele elementów maszyn wykonanych z laminatów polimerowych poddawanych jest wibracjom o dużej intensywności. Z uwagi na lepkosprężystą naturę tego typu kompozytów podczas ich drgań można zaobserwować specyficzne charakterystyczne dla nich efekty, takie jak np. dyssypacja energii. Dlatego konieczne jest zbadanie i zrozumienie ich zachowania przy długotrwałych wymuszeniach dynamicznych oraz opracowanie odpowiednich modeli, a także metod pozwalających w skuteczny sposób monitorować i diagnozować ich stan techniczny. W niniejszej pracy autorzy przedstawiają wyniki badań eksperymentalnych polimerowych płyt prostokątnych zbrojonych włóknem szklanym, dotyczących częstotliwościowej identyfikacji charakterystyk wybranych parametrów dynamicznych podczas wymuszeń rezonansowych. Badania składały się z dwóch etapów. Pierwszy etap polegał na identyfikacji funkcji odpowiedzi częstotliwościowych dla próbek o różnych długościach. Pozwoliło to na określenie częstotliwości własnych drgań próbek. W drugim etapie próbki były pobudzane do drgań rezonansowych o częstotliwościach odpowiadających pierwszym trzem częstościom giętych drgań własnych. W trakcie badań pozyskano sygnały pozwalające wyznaczyć krzywe histerezy w funkcji czasu wymuszenia. Na potrzeby badań stosowano układ pomiarowy, w skład którego wchodziły m.in. wibrometr laserowy i czujnik siły. Na podstawie zmierzonych sygnałów wyznaczono obwiednie wartości szczytowych sygnałów siły i zastosowano je do identyfikacji modelu empirycznego. Na podstawie danych pomiarowych przeanalizowano zmianę modułów dynamicznych. Zbadano wpływ częstotliwości wymuszenia oraz długości próbek na ich zachowanie dynamiczne. Na podstawie uzyskanych wyników stwierdzono, że zaproponowany model może być skutecznie zastosowany do rozwoju modeli zmęczeniowych i wytężeniowych lepkosprężystych laminatów zbrojonych włóknem szklanym.

Słowa kluczowe: polimerowe kompozyty laminowane, odpowiedź dynamiczna, histereza, moduły dynamiczne

INTRODUCTION

Machine components made of polymeric laminated composites have several advantages such as superior mechanical properties and simultaneously low weight. Therefore, an application for them, such as in aircraft turbine blades and engine components, was found. Some components made of polymeric composites were subjected to vibrations during their operation. Excessive vibrations, especially resonant vibrations, could contribute to prolonged cyclic deformation of the structure. Due to the inelastic nature of the composite matrix, the composite reveals its hysteretic behaviour (during cyclic loading), which provides out-of-phase oscillations with an accompanying energy dissipation process. In such a situation, the stress values decrease in relation to the number of cycles. Dynamic loading is connected to the self-heating effect when part of the dissipated energy turns into the heat. Owing to the poor thermal diffusivity of polymers, the temperature rises until the structure reaches a thermal steady-state.

The most crucial parameters during dynamic loading and self-heating are the frequency and magnitude of the vibrations; however, these phenomena also depend upon the geometry and initial stiffness of the structure [10]. From a condition monitoring point of view, resonant vibrations are characterized by much higher magnitudes and could be very dangerous for a machine where composite parts were utilized. Constructions subjected to cyclic loading could reach resonant frequencies in several situations: due to mistakes in the design phase, mistakes in the manufacturing phase, improper operational conditions or the shifting of natural frequencies as a result of structural degradation. In most cases, it intensifies the degradation processes and finally leads to a breakdown of the construction. Therefore, it is important to investigate the dynamic behavior of polymeric composites in order to obtain necessary data to evaluate the analytical model of a phenomenon which could be used in the diagnostics and condition monitoring of composite structures during their operation.

The problem of the vibration of a viscoelastic composite plate was investigated in several works. Alam and Asnani [1] investigated the theoretical aspects of flexural resonant vibrations of a simply-supported composite layered plate on its fundamental frequency. The authors of [2] investigate free flexural vibration of a fiber/metal composite using impact tests and determine the complex moduli of significant structures. In [3-5] the authors describe the dynamic behaviour and self-heating of polymers in tension/compression tests. The dynamic response of composites considering the thermal effects was also described by Jeyaraj et al. [6] using modal and acoustic emission analyses. They present the finite element model based on classical lamination plate theory and validate it experimentally. The method of determining the dynamic properties based on the excitation of viscoelastic material with a mass was presented in [7]. This method could be modified for the forced flexural vibrations of a cantilever beam made of a polymeric composite.

Most of the research was carried out during tension tests [3-5]. In this article the authors show the results of experiments on the determination of dynamic behaviour of the investigated GFRP rectangular plates with various lengths under a one side bending condition. In the dynamic tests, rectangular plates were subjected to forced resonant vibrations. The multiparametric empirical model was constructed based upon approximations of the obtained results. The model allows the prediction of the evolution of the hysteretic behaviour and dynamic moduli of viscoelastic composites regarding the magnitude and frequency of the vibrations and the length of the plate. Moreover, it could be used to improve the analytical modelling of the thermoviscoelasticity of polymeric composites [12].

THEORETICAL BACKGROUND

The linear viscoelastic behaviour of a polymeric composite in general can be presented in terms of the Boltzmann superposition principle (see e.g. [13]). Such behaviour in the relaxation process could be characterized by means of several classical (e.g. Maxwell model, Voight model, Zener model) and non-classical (e.g. Scott-Blair model, fractional Maxwell model, Maxwell--Weichert model) rheological models [8]. Most commonly the stress relaxation process is described using the Maxwell model [9]:

$$\sigma(t) = \varepsilon(t)E_0 - \int_0^t \varepsilon(\tau)R(t-\tau)\mathrm{d}\tau \tag{1}$$

where $\sigma(t)$ and $\varepsilon(t)$ are the time-dependent stress and strain respectively, E_0 is the instantaneous Young's modulus, τ is the relaxation time and $R(t\text{-}\tau)$ is the relaxation kernel described as follows:

$$R(t-\tau) = -\frac{\partial E(t-\tau)}{\partial t} \tag{2}$$

However, the Maxwell model does not properly consider the rapid drop in stress observed during the stress relaxation process. Therefore, it is necessary to apply its modified form. For considering such phenomena it is obvious to use the Maxwell-Weichert model, which is the generalized form of the Maxwell model and consists of i Maxwell elements. Following this, we could modify (1) in the following way:

$$\sigma(t) = \varepsilon(t)E_0 - \sum_{i=1}^{n}\int_0^t \varepsilon(\tau_i)R_i(t-\tau_i)\mathrm{d}\tau_i \tag{3}$$

This model characterizes by i relaxation times τ_i, thus it has i relaxation kernels $R(t\text{-}\tau_i)$.

By assumption of the harmonic loading, the stress and strain can be presented as:

$$\sigma = \sigma_0 \exp(i\omega t + i\delta)$$
$$\varepsilon = \varepsilon_0 \exp(i\omega t) \tag{4}$$

where σ_0 and ε_0 are the stress and strain amplitudes, $\omega = 2\pi f$ is the angular velocity and δ is the phase shift.

According to the approximate formulation of the theory of viscoelasticity [10] the stress and strain could be expressed using complex parameters. The complex modulus could be obtained by dividing the stress by the strain given by (4). Complex modulus E^* can be decomposed to real part E' - the storage modulus and imaginary part E'' - the loss modulus. Considering their frequency dependence we obtain

$$E^*(i\omega) = E'(\omega) + iE''(\omega) = |E^*| \exp[i\delta(\omega)] \tag{5}$$

There is also the connection of the complex modulus with the tangent loss factor:

$$\tan\delta = E''/E' \tag{6}$$

Complex dynamic modulus could be estimated on the basis of experimental measurements of the displacement d and applied force P. Thus, by using peak-to-peak values of d and P, complex dynamic modulus can be calculated as follows:

$$E^* = \Delta P / \Delta d \tag{7}$$

where the loss factor could be determined as a time shift between zero points of ΔF and Δd:

$$\delta = \frac{\Delta t}{T} \cdot 360° \tag{8}$$

Further, the character of the evolution of dynamic behaviour and dynamic moduli will be investigated experimentally and their parameters will be determined and analyzed.

SPECIMENS AND EXPERIMENTAL SETUP

The specimens were manufactured from GFRP in the form of unidirectional preimpregnated fibers. The laminate was manufactured from epoxy E-glass prepreg PR-UD EST 250/635 FT102 35 supplied by Epo GmbH. The lay-up of the laminate given by the structural formula $[0/\pm 60/\mp 60/0]_S$ was selected to achieve transversal isotropic properties. The material properties of the single layer are as follows [11]: Young and shear moduli E_1 = 38.283 GPa, E_2 = 10.141 GPa, G_{12} = = 3.533 GPa, Poisson ratio ν_{12} = 0.366 and density ρ = 1794 kg/m^3. In order to perform the experiments, 30 specimens were prepared. The specimen widths (W) and thicknesses (H) were respectively W = 10±0.1 mm, H = 2.5±0.05 mm. In order to investigate different cases of viscoelastic behaviour of the material, the five following cases of specimen length were considered: 200, 225, 250, 275, 300 mm.

The specimens were investigated on a laboratory stand (Fig. 1) consisting of the following components:

- specimen holder *7* which enables mounting of specimen under repeatable conditions. Holder provided insulation of heat produced during self-heating effect,
- electrodynamic shaker *1* with power amplifier *10* which was driven by generator *9* with built-in signal analyzer *8*,
- impedance head *2* which provided acceleration and force signals in order to control excitation parameters as well as determine dynamic characteristics of the specimen. Impedance head was connected with specimen *5* through stinger *3* with small specimen clamp *4* at its end,
- single point laser Doppler vibrometer *6* used for rapid and non-contact measurement of specimen vibrations,
- multi-channel signal analyzer *8* and PC *11* with software designed for modal analysis, allowing recording and determination of specimen resonant characteristics.

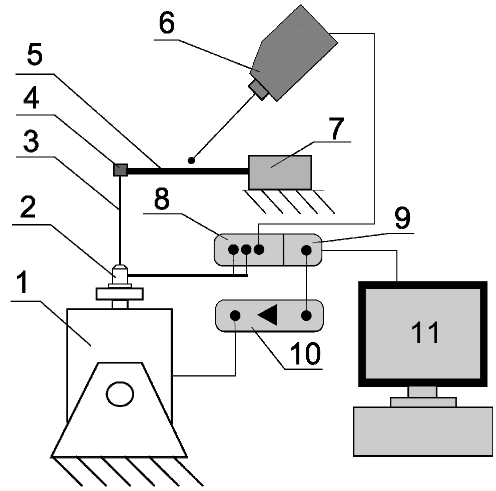

Fig. 1. Experimental setup
Rys. 1. Stanowisko badawcze

The experimental test of each specimen consisted of two steps. The primary goal of the first step was identification of the resonant frequencies of the specimen. In the second step, the specimen was excited to harmonic vibration characterized by a frequency equal to the earlier identified resonant frequency.

The experiment was administered in such way that allowed us to perform the tests for three first resonant frequencies. For each case of specimen length and each considered resonant frequency, two specimens were tested.

RESULTS AND DISCUSSION

During the modal analysis of the composite specimens, a set of Frequency Response Functions (FRF) on the basis of force and velocity signals were obtained. In order to control the quality of the FRF, estimation of the coherence (COH) functions was also computed. The exemplary FRF and COH function are presented in Figure 1. Modal analysis of the specimens indicated that bending as well as torsional modes of vibration exists. In the research, only the first three bending modes and corresponding frequencies (indicated in Fig. 1) were considered. Hereinafter the subscript ‘r’ of ‘f’ denotes the bending natural frequency and the number in the superscript denotes the number of the natural frequency. The obtained values of natural frequencies for all the specimens are tabulated in Table 1. Symbol L_e denotes the effective length of the specimen, i.e. length without clamped end ($L_e = L - 20$ mm).

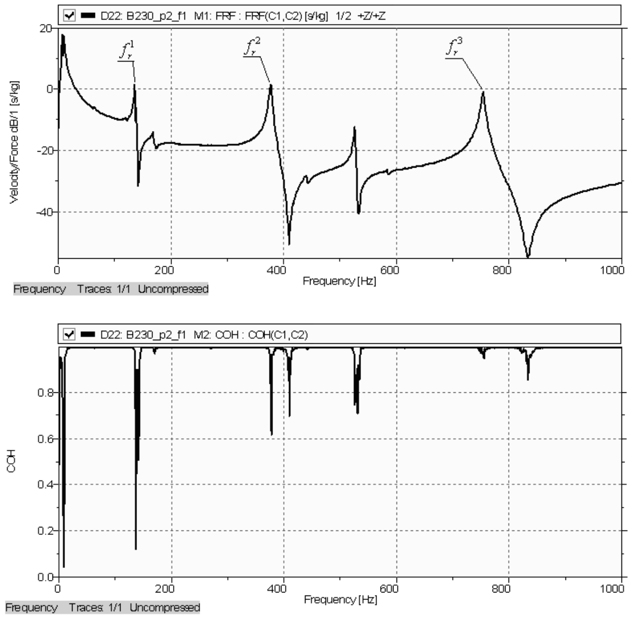

Fig.1. Exemplary FRF and COH functions for specimen with length of 250 mm

Rys.1. Częstotliwościowa funkcja przejścia i koherencja dla próbki o długości 250 mm

TABLE 1. **Natural frequencies of investigated specimens**
TABELA 1. **Częstotliwości własne badanych próbek**

L_e [mm]	180	205	230	255	280
f_r^1 [Hz]	262	161	137	128	108
f_r^2 [Hz]	733	490	425	301	235
f_r^3 [Hz]	1238	1061	836	702	480

After determination of the natural frequencies, the dynamic tests were carried out. The specimens were excited on each particular natural frequency and the applied force P, acceleration and velocity were measured during the tests. The signal velocity was at a point located close to the specimen clamp and single integration in order to obtain the displacement d signal was additional performed. On the basis of the measured signals of P and d, series of hysteresis loops across the time of observation were evaluated. An exemplary evolution of the hysteresis loop along the time of observation (the loops were presented for each 10 s) and a comparison of the hysteresis loops at the beginning and at the end of the experiment were presented in Figure 2. Area decay in the function of time could be clearly identified.

The time evolution of the hysteresis loops, determined by analysis of the curve constructed from the peak-to-peak values of $d(t)$ and $P(t)$, is presented in Figure 2. Similarly, the peak-to-peak curves were constructed for the various lengths of specimens and excitation resonant frequencies. Considering the character of the obtained curves, it is evident that they behave as exponential decay-like functions. The double-exponential model (10) was proposed (considering constant displacement rates) based upon the normalized values of the envelope from the peak values of the measured force signal (9). Such a model could be useful for estimating dependencies between the length and resonant frequency and the evolution of peak-to-peak curves in the application of analytical computation to the non-steady self-heating temperature distributions.

$$P_N(t) = \frac{P_{p-p}(t)}{\max(P_{p-p}(t))} \tag{9}$$

where $P_{p\text{-}p}(t)$ is the envelope obtained from the peak values of the force signal

$$P_N(t) = A_1 \exp(\lambda_1 t) + A_2 \exp(\lambda_2 t) \tag{10}$$

where $P_N(t)$ is the normalized force, A_i and λ_i are parameters of the model to be determined. For approximation purposes, the unconstrained nonlinear optimization algorithm, which finds the minimum of a scalar function of several variables, was applied. Figure 3 depicts exemplary force envelope curves with their approximations. In each case, the coefficient of multiple determination (R-squared) did not depreciate 98.5% and in most cases it was very close to 1.

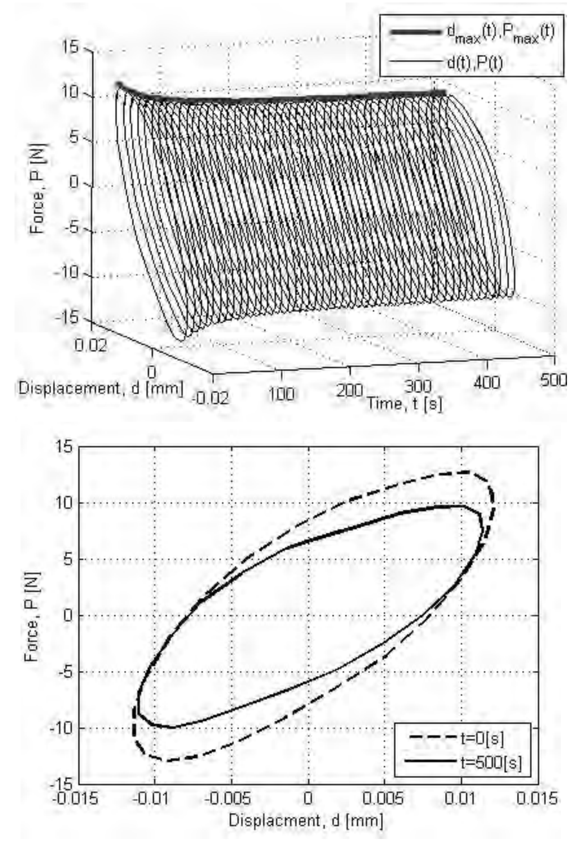

Fig. 2. Hysteresis loop evolution and hysteresis loops at the beginning and at end of loading

Rys. 2. Rozwój pętli histerezy oraz pętla histerezy na początku i na końcu obciążenia

Using (7) and (8), it is possible to determine the time evolution of the dynamic moduli. From the waveforms of $d(t)$ and $P(t)$ we could determine period T and the loss factor and then the complex modulus. The components of the complex modulus, i.e. storage and loss moduli could be determined from the following formulae:

$$E'(t) = E^*(t)\cos\delta(t), \quad E''(t) = E^*(t)\sin\delta(t) \qquad (11)$$

which could be an alternative method for determining frequency and temperature dependencies of the complex moduli to dynamic mechanical analysis.

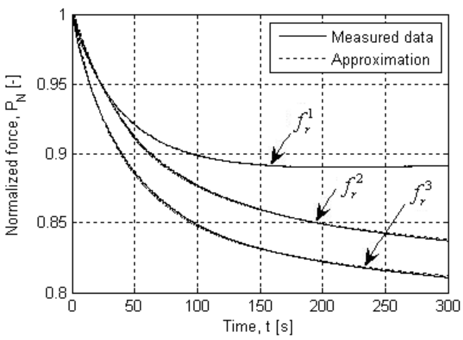

Fig. 3. Peak-to-peak force envelope curves and their approximations for specimens with length of 225 mm

Rys. 3. Krzywe obwiedni wartości szczytowych siły i ich aproksymacje dla próbek o długości 225 mm

Using (10), it was possible to determine the parameters of the approximation function. Table 2 presents the values of these parameters depending on the specimen length and excitation frequency. Such dependencies could be observed also during DMA (Dynamic Mechanical Analyzer) tests.

TABLE 2. **Approximation parameters of empirical model of force descent during dynamic test**

TABELA 2. **Parametry empirycznego modelu aproksymacyjnego spadku wartości siły przy teście dynamicznym**

L_e [mm]	R [-]	A_1 [-]	λ_1 [1/s]	A_2 [-]	λ_2 [1/s]
180	1	0.2262	-0.0157	0.7728	$-4.048\cdot10^{-4}$
	2	0.1308	-0.0879	0.8968	$-3.807\cdot10^{-4}$
	3	0.1059	-0.0265	0.8913	$-2.209\cdot10^{-4}$
205	1	0.1652	-0.0221	0.8317	$-7.142\cdot10^{-5}$
	2	0.2049	-0.0152	0.8112	$-3.867\cdot10^{-5}$
	3	0.1492	-0.0229	0.8452	$-1.434\cdot10^{-4}$
230	1	0.2129	-0.0237	0.7827	$-1.169\cdot10^{-4}$
	2	0.1747	-0.0218	0.8258	$-7.494\cdot10^{-5}$
	3	0.1224	-0.0129	0.8859	$-1.298\cdot10^{-4}$
255	1	0.2074	-0.0229	0.7889	$-1.046\cdot10^{-4}$
	2	0.2122	-0.0169	0.7876	$-1.044\cdot10^{-4}$
	3	0.2106	-0.0372	0.8441	$-1.216\cdot10^{-4}$
280	1	0.2236	-0.0225	0.7684	$-1.109\cdot10^{-4}$
	2	0.1816	-0.0209	0.8150	$-1.250\cdot10^{-4}$
	3	0.1042	-0.0182	0.8952	$-1.058\cdot10^{-4}$

Exemplary 3D-plots of A_1 and λ_1 for f_r^2 are presented in Figures 4 and 5, respectively. The approximation parameters depended on the specimen length and its natural frequency presented as the 2D contour plot projections in Figures 6-9, where the grey levels denote the values of the given parameters.

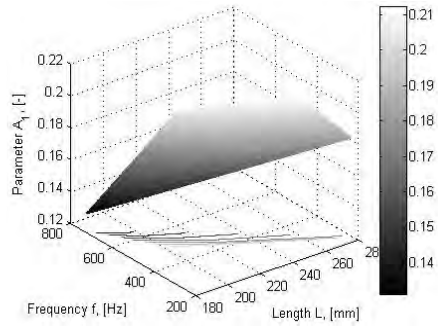

Fig. 4. 3D-plot of the parameter A_1 for f_r^2

Rys. 4. Wykres 3D parametru A_1 dla f_r^2

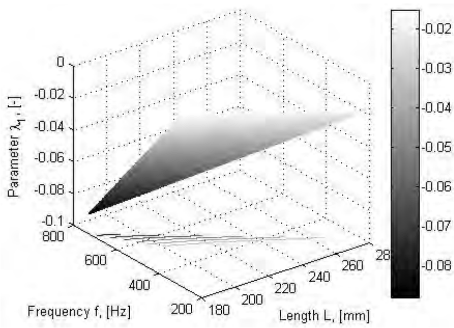

Fig. 5. 3D-plot of the parameter λ_1 for f_r^2

Rys. 5. Wykres 3D parametru λ_1 dla f_r^2

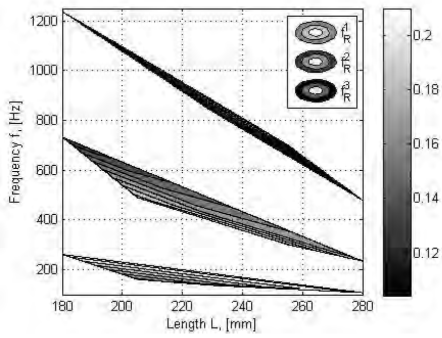

Fig. 6. Variability of the parameter A_1

Rys. 6. Zmienność parametru A_1

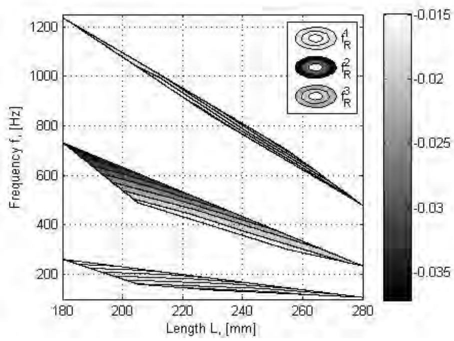

Fig. 7. Variability of the parameter λ_1

Rys. 7. Zmienność parametru λ_1

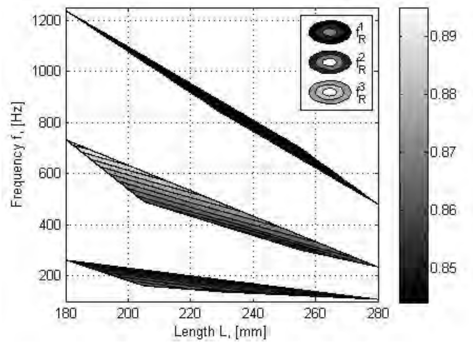

Fig. 8. Variability of the parameter A_2

Rys. 8. Zmienność parametru A_2

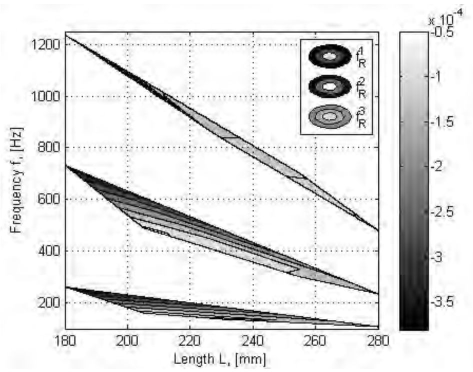

Fig. 9. Variability of the parameter λ_2

Rys. 9. Zmienność parametru λ_2.

CONCLUSIONS

In the following study, the structural behaviour of polymeric laminated composites subjected to cyclic load was presented. The specimens were excited on resonant frequencies with a constant displacement rate. During the analysis of the obtained dependencies it was noticed that:

- increase of excitation frequency caused decrease of first pre-exponential factor A_1 (which happened generally by decreasing the storage modulus) and increase of second pre-exponential factor A_2 (due to the increase of loss modulus),
- parameter λ_i of an exponential model decreases with increase of excitation frequency (inaccuracies of obtained results may have been caused by experimental conditions and/or assumed fitting algorithm),
- the increase of specimen length caused decrease of all four parameters of model
- all of parameters in proposed model revealed linear dependencies to various frequencies and specimen lengths.

In Figures 5-8, one can observe partial surfaces of the model parameters. In order to identify the whole model while taking into consideration the material behavior beyond resonant frequencies, additional research is necessary. The obtained model allows prediction of the hysteresis evolution of viscoelastic plates during cyclic loading which coincided with energy dissipation and the self-heating effect. Moreover, it is possible to determine the complex dynamic modulus and its components and to predict their time evolution using the presented measurement procedure and fitting model. In further research, the connection of the determined parameters with the self-heating will be showed. The understanding and proper description of the dynamic behaviour of such structures allows the use of the presented results in further investigations in the area of the description of fatigue mechanisms with regard to self-heating.

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