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COMPOSITES THEORY AND PRACTICE

Krzysztof Majerski*, Barbara Surowska, Jarosław Bieniaś

Lublin University of Technology, Faculty of Mechanical Engineering, Department of Materials Engineering, ul. Nadbystrzycka 36, 20-618 Lublin, Poland
**Corresponding author: e-mail: k.majerski@pollub.pl*

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NUMERICAL MODELING OF LOW-CYCLE FATIGUE OF FIBER REINFORCED COMPOSITE

The aim of this paper is to present the adoption of the progressive damage model to describe the degradation of polymer composites under ultra-low-cycle fatigue. The first part of the paper is devoted to the presentation of the approach and discussion of the theoretical aspects of the numerical model. The model contains three states describing the material degradation process: undamaged response, point of damage initiation and damage evolution. The second part is devoted to the implementation of the presented model to describe the fatigue life of the polymer composite under ultra-low cycle fatigue. A polymer composite comprising unidirectional layers was modeled as linear elastic with progressive stiffness degradation. The model was loaded in cyclic tension with a sinusoidal waveform. Numerical modeling has been performed using Abaqus software. The results of the numerical analysis results were compared with the experimental results taken from literature.

Keywords: **polymer composite, numerical modeling, low-cycle fatigue, progressive degradation**

MODELOWANIE ZMĘCZENIA NISKOCYKLOWEGO KOMPOZYTU WŁÓKNISTEGO

Celem pracy jest wskazanie na możliwość implementacji progresywnego modelu zniszczenia do opisu degradacji kompozytu polimerowego wzmacnianego włóknami szklanymi w różnych konfiguracjach w warunkach zmęczenia niskocyklowego o bardzo małej liczbie cykli. W pierwszej części pracy nakreślono ważniejsze aspekty związane z modelowaniem polimerowego kompozytu włóknistego z wykorzystaniem progresywnego modelu zniszczenia zaproponowanego przez Z. Hashina. Opis stanu kompozytu zawiera odpowiedź materiału w stanie wyjściowym oraz identyfikuje zjawiska inicjacji i rozwoju zniszczenia w objętości laminatu. Druga część pracy poświęcona jest próbie implementacji modelu w środowisku metody elementów skończonych. Do modelowania degradacji kompozytu pod działaniem cyklicznych obciążeń podkrytycznych o wartości zbliżonej do wytrzymałości statycznej wykorzystano oprogramowanie Abaqus. Analizę statyczną oraz niskocyklową przeprowadzono dla laminatów złożonych z warstw jednokierunkowych o układach: [0], $[0,90]_s$, $[\pm45]_{4s}$. Podczas analizy wykorzystano modele próbek kompozytowych o geometrii zgodnej z normą ASTM 3039D. W pierwszym etapie przeprowadzono próbę statyczną w celu weryfikacji modelu. Następnie model poddano cyklicznym obciążeniom sterowanym przemieszczeniem o wartościach odpowiadających: 0,98; 0,96; 0,94; 0,92; 0,9 poziomu naprężeń niszczących w próbie statycznej dla odpowiedniego układu warstw. W przeprowadzonych analizach uzyskano zadowalającą zgodność wyników modelowania z eksperymentami. Wskazano przebiegi funkcji poszczególnych parametrów opisujących zniszczenie komponentów laminatów w zależności od orientacji warstw wzmacniających.

Słowa kluczowe: **kompozyty polimerowe, modelowanie numeryczne, zmęczenie niskocyklowe, model progresywnego zniszczenia**

INTRODUCTION

Polymer composites reinforced by fibres are advanced engineering materials having a wide range of applications in the aerospace industry. Materials of this type have been successfully used for primary and secondary aircraft structures [1, 2]. One of the fundamental engineering problems associated with the use of polymer composites is to determine their sustainability in terms of the occurrence of complex cyclic loading conditions. Despite the numerous advantages such as high specific strength and stiffness-to-density ratio, fiber reinforced composites are strongly anisotropic materials, in which the degradation process occurs in a complex manner.

Describing the fatigue damage of composite materials subjected to repeated loadings is complicated. Unlike alloys which are homogeneous, predicting the fatigue damage of these structures is focused on observations of the growth of a single dominant crack which is the major cause of ultimate failure [3]. The fatigue mechanisms occurring in composite materials involve the nature of the microstructural diversity of the material volume and interfacial effects. Degradation processes in ultra-low cycle fatigue are very dynamic. The type of damage is correlated with the type of reinforced fibers and their orientations, type of matrix, stacking sequence, environmental conditions and type of loading. Major fatigue models and life time prediction approaches belong to three main groups: fatigue models based on S-N curves, phenomenological models for residual strength/stiffness and progressive damage models [4, 5].

PROGRESSIVE DAMAGE MODEL

Progressive damage models are characterized by the introduction of one or more variables describing the degradation of the composite material. The specific damage mode is in direct relation to the proposed evolution law. Damage description is conducted on a micro scale, however, damage accumulation leads directly to macroscopic reduction of the mechanical properties. Progressive models can be divided into two basic classes: models that correlate an increase in the damage function and residual strength/stiffness and those whose task is to predict damage growth such as delamination size and crack volume density. A detailed comparison of the various aspects of progressive damage model assumptions is presented in [6].

Failure criteria

The degradation of composite material subjected to cyclic loadings can be expressed as the accumulation of micro-cracks occurring in the material volume, which reduce the effective cross-section of the sample. According to the approach proposed in [6, 7], damage can be defined as a scalar function in the range of <0.1> where: 0 - means an undamaged state and 1 - ultimate failure. The evolution law of these scalar functions called damage variables is related to fracture energy G_c. The damage is described in space effective stress because only the residual area is involved in the stress transfer.

The result of the damage process is the degradation of the stiffness matrix that has the form

$$Q=\frac{1}{D}\begin{bmatrix} (1-d_f)E_1 & (1-d_f)(1-d_m)\nu_{21}E_1 & 0 \\ (1-d_f)(1-d_m)\nu_{12}E_2 & (1-d_m)E_2 & 0 \\ 0 & 0 & D(1-d_s)G_{12} \end{bmatrix} \tag{1}$$

where $D = 1 - (1 - d_f)(1 - D_m)\nu_{12}\nu_{21}$, d_f, d_m, and d_s are the damage variables for the fibers, matrix and share failure modes and E_1, E_2, G_{12} are the undamaged material moduli respectively. It is very important that the damage variables for the fibers and matrix have different values for tension and compression. This anisotropic damage model is addressed in plane stress formulation and the damage variable responsible for shear is not independent.

Damage initiation

Failure initiation is defined as the point at which the process of irreversible degradation of the composite material begins.

To accurately reflect the degradation characteristics, it is necessary to use four independent damage initiation functions proposed by Z. Hashin [6, 8]:

Fiber tension $(\hat{\sigma}_{11} \geq 0)$:

$$F_{ft} = \left(\frac{\hat{\sigma}_{11}}{X^T}\right)^2 + \alpha\left(\frac{\hat{\sigma}_{12}}{S^L}\right)^2 = 1 \tag{2}$$

Fiber compression $(\hat{\sigma}_{11} < 0)$:

$$F_{fC} = \left(\frac{\hat{\sigma}_{11}}{X^C}\right)^2 = 1 \tag{3}$$

Matrix tension $(\hat{\sigma}_{22} \geq 0)$:

$$F_{mt} = \left(\frac{\hat{\sigma}_{22}}{Y^T}\right)^2 \left(\frac{\hat{\sigma}_{12}}{S^L}\right)^2 = 1 \tag{4}$$

Matrix compression $(\hat{\sigma}_{22} < 0)$:

$$F_{mt} = \left(\frac{\hat{\sigma}_{22}}{2S^T}\right)^2 + \left[\left(\frac{Y^C}{2S^T}\right)^2 - 1\right]\frac{\hat{\sigma}_{22}}{Y^C} + \left(\frac{\hat{\sigma}_{12}}{S^L}\right)^2 = 1 \tag{5}$$

where:
- $\hat{\sigma}_{ij}$ - represents the effective stress tensor components,
- X^T, X^C - tensile and compressive strength respectively in the fibers direction,
- Y^T, Y^C - tensile and compressive strength respectively, transverse to fibers direction,
- S^L, S^T - longitudinal and transverse shear strengths respectively,
- α - effect of shear stress in fiber tensile initiation criterion.

Fulfillment of one of the above criteria results in the appearance of positive values of the damage variables that control the decrease in stiffness coefficients.

Damage evolution

Damage evolution is controlled directly by fracture energy dissipation G_c for each damage variable. In practice this means that for a constant increase in strain, we observe a constant increase in stress (which is proportional to the initial stiffness of the material) to the point where the damage initiation criterion is met (ε_0) after which we observe a linear decrease in the stress caused by the degradation of material stiffness. This relation is shown in Figure 1.

The fracture energy dissipated during damage progression is proportional to the volume of the damaged material. For this reason, to ensure that the model is useful for the finite element method, Bazant and Oh in [10] proposed the use of the characteristic element length (L_C). This approach prevents strong reduction in the energy dissipated along with mesh refinement.

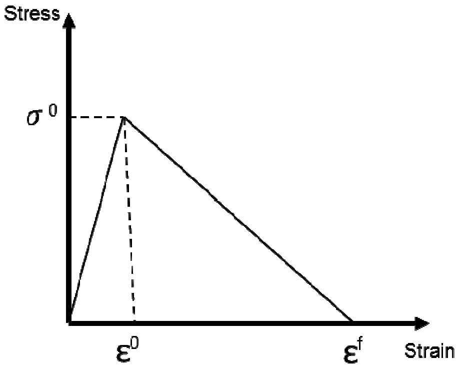

Fig. 1. Stress vs strain relation for material with linear softening

Rys. 1. Krzywa naprężenie-odkształcenia dla materiału z liniowym osłabieniem

Failure strain, which represents the strength limit, is related to fracture energy in the following way:

$$\varepsilon^f = \frac{2G^C}{\sigma^0 L^C} \tag{6}$$

where:

ε^f - failure strain,

G^C - fracture energy,

σ^0 - stress at initiation point,

L^C - characteristic length of the element.

The values of failure strain for the damage variables are controlled by the fracture energy determined experimentally for each type of damage. For the purposes of implementing the model for numerical calculation, the evolution of the damage variables is expressed in terms of equivalent stress and displacement, and the energy dissipated during the damage process is related to adequate fracture energy. Computation of the damage variables is performed using equivalent displacement. The equivalent displacement and the corresponding equivalent stress are calculated based on equations of initiation criteria for each failure mode. Assuming this destruction parameter for each i-th method/module, destruction is defined by the following equation:

$$d_1 = \frac{\delta^f_{I,eq}(\delta_{I,eq} - \delta^0_{I,eq})}{\delta_{I,eq}(\delta^f_{I,eq} - \delta^0_{I,eq})}; \quad \delta^0_{I,eq} \le \delta_{I,eq} \le \delta^f_{I,eq}$$

$$I \in \{ft, fc, mt, mc\} \tag{7}$$

The $\delta^0_{I,eq}$ is defined here as the equivalent displacement at which the initiation criterion is reached, and $\delta^f_{I,eq}$ corresponds to the equivalent displacement at which the material is completely damaged. In order to calculate $\delta^f_{I,eq}$, we use the following relation:

$$\delta^f_{I,eq} = \frac{2G^C_I}{\sigma^0_{I,eq}} \tag{8}$$

where $\sigma^0_{I,eq}$ is the equivalent stress at which the initiation criterion is met, on the assumption that softening of the material is linear. The definitions of equivalent displacements and equivalent stresses are connected with a specific failure mode and are listed in Table 1.

TABLE 1. **Definitions of equivalent stress displacements and scaling functions**

TABELA 1. **Definicje równoważnego naprężania przemieszczeń i funkcji skalowania**

Failure mode	$\boldsymbol{\delta_{eq}}$	$\boldsymbol{\sigma_{eq}}$	**Scaling functions**
Fiber tension ($\hat{\sigma}_{11} \ge 0$)	$L_C\sqrt{\langle\varepsilon_{11}\rangle^2 + \alpha\varepsilon_{12}^2}$	$\frac{L_C(\langle\sigma_{11}\rangle\langle\varepsilon_{11}\rangle + \alpha\sigma_{12}\varepsilon_{12})}{\delta^{ft}_{eq}}$	$\frac{1}{\sqrt{F^{ft}}}$
Fiber compression ($\hat{\sigma}_{11} < 0$)	$L_C\langle-\varepsilon_{11}\rangle$	$\frac{L_C\langle-\sigma_{11}\rangle\langle\varepsilon_{11}\rangle}{\delta^{ft}_{eq}}$	$\frac{1}{\sqrt{F^{fc}}}$
Matrix tension ($\hat{\sigma}_{22} \ge 0$)	$L_C\sqrt{\langle\varepsilon_{22}\rangle^2 + \varepsilon_{12}^2}$	$\frac{L_C(\langle\sigma_{22}\rangle\langle\varepsilon_{22}\rangle + \sigma_{12}\varepsilon_{12})}{\delta^{mt}_{eq}}$	$\frac{1}{\sqrt{F^{mt}}}$
Matrix compression ($\hat{\sigma}_{22} < 0$)	$L_C\sqrt{\langle-\varepsilon_{22}\rangle^2 + \varepsilon_{12}^2}$	$\frac{L_C(\langle-\sigma_{22}\rangle\langle-\varepsilon_{22}\rangle + \sigma_{12}\varepsilon_{12})}{\delta^{mc}_{eq}}$	$\frac{-y+\sqrt{\alpha^2+4\beta}}{2\beta}$ where: $y=\left[\left(\frac{Y^C}{2S^T}\right)-1\right]\frac{\hat{\sigma}_{22}}{Y^C}$ $\beta=\left(\frac{\hat{\sigma}_{22}}{2S^T}\right)^2+\left(\frac{\hat{\sigma}_{12}}{S^L}\right)$

The equivalent displacement and stress in the post initiation phase are calculated using multiplication of its value by an adequate scaling factor. The scaling factors for each considered failure mode are listed in Table 1. In uniaxial loading conditions, such a function selection provides a direct relation between the energy dissipated during damage and adequate fracture energy G_{Ic}.

NUMERICAL IMPLEMENTATION

To introduce the above-presented progressive damage model to the finite element method environment, Abaqus software was employed.

In the present study, the damage of a glass-fiber-reinforced polymer composite under ultra-low-cycle fatigue was modeled as a series of sub-critical static steps of tension loading and unloading in a sinusoidal manner. The Newton-Raphson method was used to solve systems of nonlinear equations. 8-node hexa hedron, continuum shell - SC8R elements were used to model individual composite layers. Models of composite samples were made according to the ASTM standard 3039D with a suitable mesh density (element length 0.25 mm). Cuboidal sample models were built in such a manner that one layer of stacked elements corresponds to one element layer of the composite. The orientation of the elements remained consistent with the actual fiber arrangement. The sample models were equipped with protective end tabs with a thickness of about 1 mm and length of 50 mm, modeled as a linearly elastic isotropic material, with a Young's modulus of 73 GPa. Displacement was applied on the tab surfaces while the second pair of end tabs was fixed.

The progressive damage model was applied to describe the degradation of the glass/epoxy unidirectional reinforced composite. The configuration of the modeled laminates is described in Table 2.

TABLE 2. **Arrangement of modeled laminates**
TABELA 2. **Ułożenie modelowanych laminatów**

Material	**Number of layers**	**Layer thickness [mm]**	**Laminate configurations**
GFRP/epoxy	4	0,25	[0]
GFRP/epoxy	4	0,25	$[0,90]_s$
GFRP/epoxy	16	0,25	$[\pm45]_{4s}$

The material properties used for modeling were carefully chosen and listed in Table 3.

The first step of the analysis was to perform the modeling of the static tensile test for comparison with the experiment and calibration of the model. The results of the numerical calculations are presented in Figure 2.

TABLE 3. **Material properties of analyzed laminates**
TABELA 3. **Właściwości materiałowe badanych laminatów**

GFRP/epoxy	
E_1 [MPa]	56000
E_2 [MPa]	16000
$G_{12} = G_{13}$ [MPa]	5500
$G_{12} = G_{13}$ [MPa]	3000
ν_{12}	0.33
X^T [MPa]	1560
Damage initiation properties	
X^C [MPa]	1300
Y^T [MPa]	55
Y^C [MPa]	214
S^L [MPa]	67
G_{ft} [N/mm²]	12.5
G_{fc} [N/mm²]	12.5
G_{mt} [N/mm²]	1
G_{mc} [N/mm²]	1

In the static test, the modeling succeeded in receiving a satisfactory correlation between the analysis results and the experiment. The essential differences between the shapes of the individual curves for different layer orientations are clearly visible.

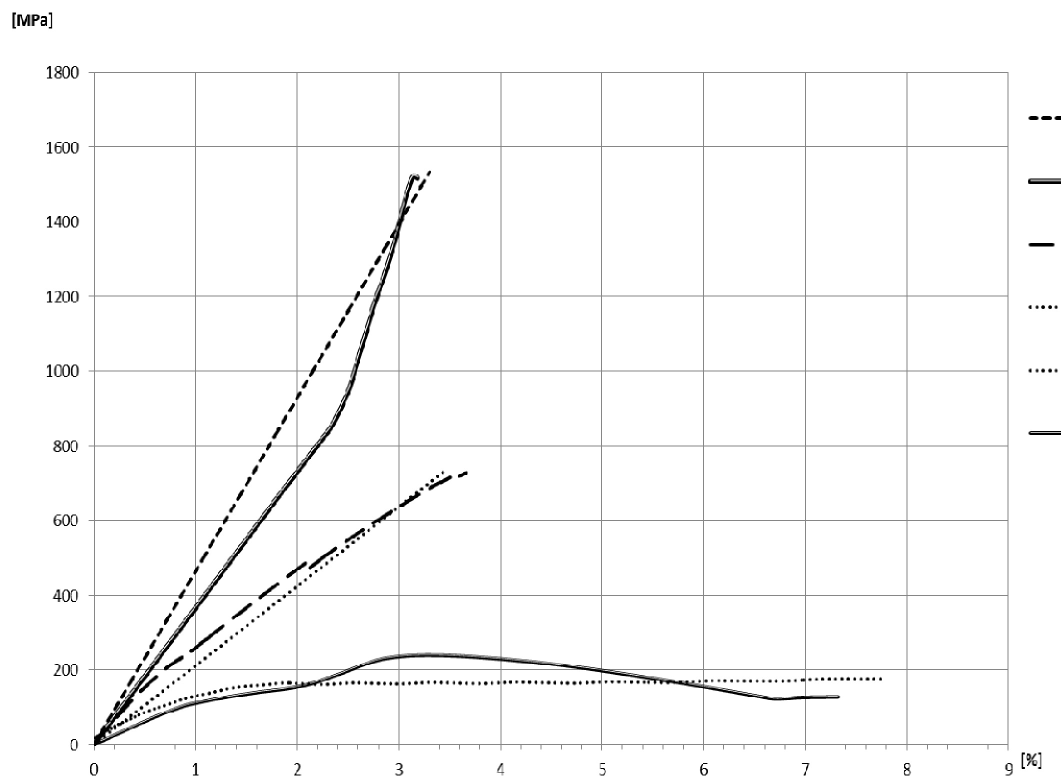

Fig. 2. Comparison of stress/strain curves for laminates with different fiber orientations obtained in experiments and numerical analysis

Rys. 2. Porównanie krzywych naprężenie-odkształcenie dla laminatów w różnej konfiguracji - wyniki eksperymentalne oraz uzyskane podczas analizy numerycznej

The second part of the analysis was focused on the modeling of low-cycle fatigue. Due to the level of fracture energy and dynamic nature of material degradation, it was necessary to use a damage function stabilization parameter, but the size of this parameter was set in the range of 0.004÷0.006 so that the energy generated as a result of stabilization was incomparably smaller than the energy of the whole model. The thermal effects have not been taken into consideration in this study.

During the analysis, the modeled samples were loaded by displacement with a value corresponding to five levels of maximal load: 0.98, 0.96, 0.94, 0.92, 0.9. The test duration was 100 loading cycles. The test was interrupted when any of the damage variables reached a value of 1. If none of the damage variables reached the value of 1 after 100 cycles the simulation was terminated.

The results for the analyzed lay-ups have been compiled with the results of the low-cycle fatigue experiments taken from [11].

The results of the numerical analysis in some cases show satisfactory compliance with experiments but do not in others. Firstly, all of the authors emphasize that in the case of low-cycle fatigue of fiber-reinforced polymer composites, a large scatter of experiment results occurs. This is due to the complexity of the internal structure and its diversity on the microscale. Secondly, failure in the different lay-up arrangements occurs as a result of other types of damage. Figure 4 shows the course of representative examples of damage variables for respective lay-up arrangements.

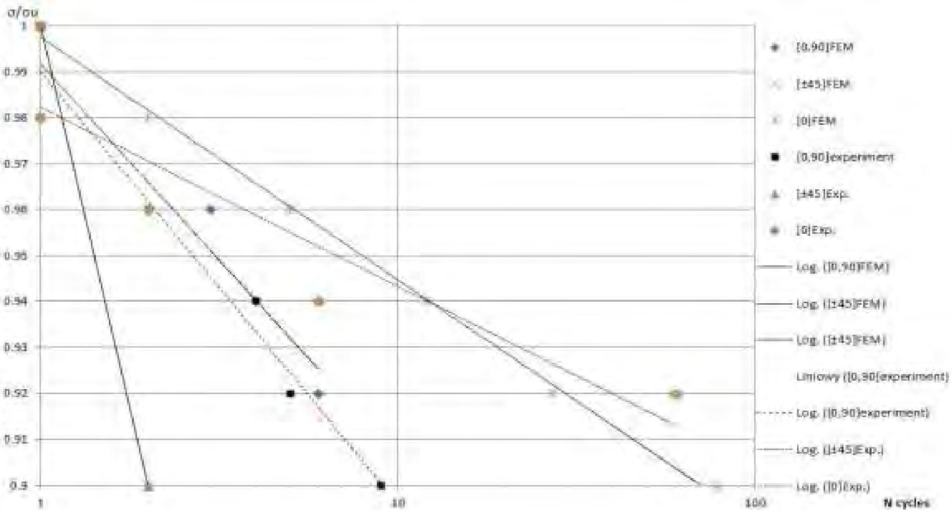

Fig. 3. Results of low cycle fatigue modelling compared with those obtained in experiments

Rys. 3. Wyniki modelowania zmęczenia niskocyklowego zestawione z wynikami eksperymentów

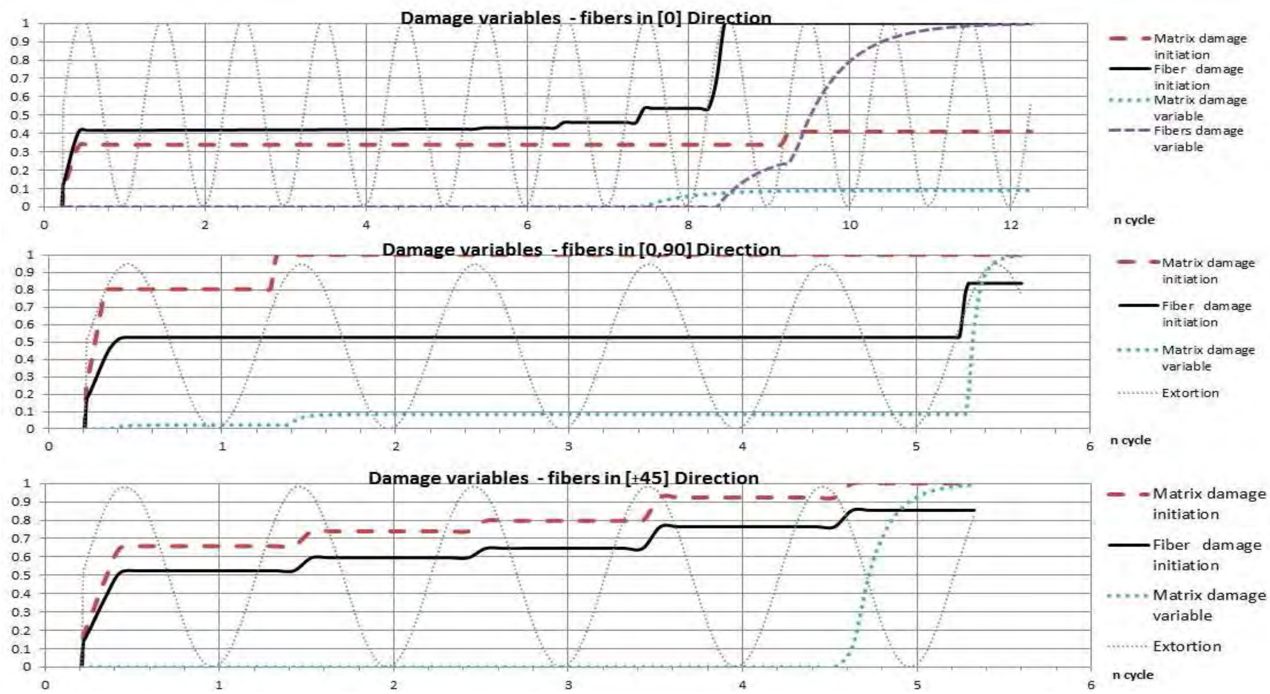

Fig. 4. Course of representative example of damage variables for respective lay-up arrangements

Rys. 4. Przebieg reprezentatywnych zmiennych funkcji opisujących proces degradacji modelowanych laminatów w różnych konfiguracjach

The received course of damage variables indicate a dominant failure character in each specific case. The dominant failure modes obtained for all the considered systems are consistent with the results of the experimental observations. Failure of the laminate arranged in the 0 direction occurred through the accumulation of fiber defects. The failure of the [±45] laminate occurred due to matrix destruction (interlaminar shear). The [0,90] laminate was damaged through destruction of the matrix which lead to an increase in the fibre damage variable and total failure.

CONCLUSIONS

In this paper the progressive damage model is employed to describe the degradation of GFRP under ultra-low cycle fatigue. The modeling of composite behavior was carried out based on the finite element method. The static analysis results obtain a good correlation with the experiment. The low-cycle fatigue analysis results reveal the different failure modes for each of the lay-up configurations. Because of the lack of experimental results, the simulations are compared with those of [11]. The number of cycles to failure obtained in the analysis for the respective systems reflects the trends presented in the referenced test results, however, do not demonstrate satisfactory compliance. Other weaknesses of the model are its deterministic nature as well as its considerable complexity in adapting the calculation of large components. The obtained identification of dominant failure modes are satisfactory, however, this type of computing is very time consuming because of the full description of each load step. The results of this analysis will be used for modeling composite layers in modelling the low cycle fatigue of fiber-metal laminates.

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