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COMPOSITES THEORY AND PRACTICE

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THE APPLICATION OF FUZZY LOGIC IN FATIGUE DAMAGE ANALYSIS

In this study, fatigue damage progress is analyzed both theoretically and experimentally. Cyclic loading causes damage, reducing the strength until the material can no longer sustain even service loading. The theoretical analysis is associated with the definition of the damage parameter. The experimental analysis is mainly devoted to the consideration of two structural elements, i.e. a rectangular composite plate (made of glass fibre/epoxy resin) with a centrally located circular hole subjected to cyclic tensile stress and a square plate (made of aramid fiber/epoxy resin) subjected to shear loading. The experiments demonstrate scattering of the results. Fuzzy set analysis has been proposed in order to estimate the uncertainty in evaluation of the critical number of cycles corresponding to the final fatigue damage.

Keywords: fatigue analysis, fuzzy logic, composite plates

ZASTOSOWANIE LOGIKI ROZMYTEJ W ANALIZIE USZKODZEŃ ZMĘCZENIOWYCH

Przedstawiono teoretyczną i eksperymentalną analizę rozwoju uszkodzeń zmęczeniowych. Cykliczne obciążenie powoduje uszkodzenia, co zmniejsza wytrzymałość aż do momentu, gdy materiał nie może przenosić nawet obciążenia eksploatacyjnego. Analiza teoretyczna jest związana z definicją tzw. parametru uszkodzeń. Badania dotyczą dwóch elementów konstrukcyjnych: prostokątnych kompozytowych płyt (wykonanych z włókna szklanego/żywicy epoksydowej) z centralnie umieszczonym kołowym otworem poddanych cyklicznemu rozciąganiu oraz kwadratowych płyt (wykonanych z włókna aramidowego/żywicy epoksydowej) poddanych ścinaniu. Eksperyment wykazuje znaczny rozrzut wyników. Zaproponowano zastosowanie teorii zbiorów rozmytych do oszacowania niepewności w ocenie krytycznej liczby cykli odpowiadającej zmęczeniowemu zniszczeniu.

Słowa kluczowe: analiza zmęczeniowa, logika rozmyta, płyty kompozytowe

INTRODUCTION

A great number of monographs and review papers present the damage mechanisms in composites under fatigue conditions. In this context, it is worth mentioning monographs [1-3] and review paper [4]. Experimental data, especially for fatigue tests, have ranges of scatter affected by variability in the material microstructure from one test specimen to another, of course, if we do not want to also mention the effects of stacking sequences etc. Typically, the variability in material parameters makes it difficult to accurately predict the response of structural components and significantly affects the reliability of designs see e.g. monograph [1]. A possible scatter of experimental results is represented e.g. in paper [5].

There is a variety of failure modes associated with static and/or fatigue damage, including: 1) matrix cracking, 2) interfacial debonding, 3) fibre breakage, and 4) delaminations. The first three mechanisms characterize the local (micromechanical) behavior of composites, whereas the last one can be described with the use of meso-modelling. Building a comprehensive, local model can only be accomplished by incorporating: (i) fiber mechanical properties and flaw statistics on the length scale of local fiber load transfer, including any stochastic stress-time dependency, (ii) matrix plasticity and creep in shear around fiber breaks, (iii) interface debonding and viscous frictional sliding in terms of local shear stress, (iv) local fiber and matrix packing geometry (2-D planar, 3-D hexagonal and random) and associated residual stresses from processing, (v) multiple matrix cracking as it affects the penetration of the environment to the fiber surface. Reifsnider, Case [6], using micrographs, have evidently demonstrated the microstructural randomness of manufacture-induced defects, microvoids, fibre rupture, kinks, etc., so that a numerical analysis can always be conducted for a specific and individual construction but with a high computational cost.

Experimental data always represent a various scatter of results (e.g. [5]) and in this way the imprecision of the measured values in experimental studies in our approach are treated as fuzzy numbers. We have proposed to use fuzzy set methodology [7-10]. It is possible to construct a membership function for the measured value. We intend to adjust to the membership function representation. With the use of the above-mentioned construction, it is possible to build a Fuzzy Knowledge Base or probabilities for a given set of experimental data.

FATIGUE DAMAGE MODELING

In this paper, a one-dimensional residual stiffness model is proposed. It aims at simulating the three stages of stiffness degradation, including final failure. To that purpose, the damage evolution law consists of two terms, separately accounting for damage initiation and propagation. Macroscopic damage variable d is a macroscopic measure for fatigue damage, since structural changes on the microscopic scale are characterized by a macroscopic reduction in the stiffness. To simulate the stage of final failure, the strength properties of the composite material must be included. Therefore, a new stress measure, the fatigue failure index, has been defined, based on modified use of the classical Tsai-Wu static failure criterion:

$$RB = F_{xx}\sigma_1^2 + 2F_{xx}\sigma_1\sigma_2 + F_{yy}\sigma_2^2 + F_{ss}\sigma_6^2 + F_x\sigma_1 + F_y\sigma_2 - 1 \tag{1}$$

where σ denotes the components of the stress tensor, and F the appropriate failure indices. The fatigue failure index, defined as:

$$d = \frac{1}{RB} \tag{2}$$

is particularly useful to model the stage of final failure. This is to be introduced with values between zero (virgin material state) and unity (final mode of failure). The damage parameter characterizes and simulates the three stages of stiffness degradation (sharp initial decline - gradual deterioration - final failure) [9]. The above approach will facilitate damage (fracture) analyses to be conducted for individual plies in any arbitrarily laid-up laminate. To put it differently, this novel approach will allow one to utilize the meso-model whereby existing FE modeling can be used.

In the proposed research, the spatial non-uniformity of a material properties at the microscopic level is to be taken into account from experimental data obtained during fatigue tests conducted for plies oriented at 0°, 45° and 90° in tension and compression. When processed, this information will be represented by the lower and upper boundaries of the stiffness degradation, i.e., as stiffness $E(n)$ versus the number of cycles n relationships. These sets of lower and upper boundaries will be available independently for 0°, 45° and 90° orientations and Figures 1 and 2 illustrate the form of diagrams obtained for fibres oriented at 0° for specimens subjected to tension.

Using the arbitrary FE package, it is possible to compute the stress distribution for given material constants and further, it is possible to find the appropriate values of failure index d - Eq. (2). With the aid of fatigue diagrams characterizing the stiffness degradation for each number of n cycles, it is possible to find the distributions of failure indices with the number of cycles, i.e. $d(n)$ and determine fatigue life N_f as the failure index reaches the value of 1 (final damage). The numerical calculations are conducted for the mean, upper and lower values of stiffnesses and for the chosen increments of the cycles. It is also worth emphasizing that the numerical analysis is not limited to laminates made of plies oriented at 0°, 45° and 90° since stiffnesses for other orientations can be computed with the help of classical transformation law from a local to global system of coordinates, assuming the validity of the linear, elastic Hook relations for orthotropic materials.

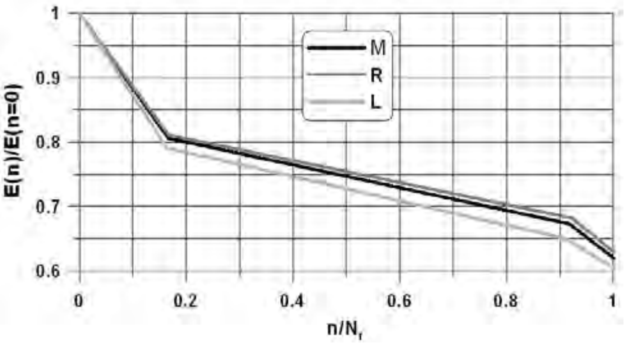

Fig. 1. Degradation of laminate (glass fiber/epoxy resin) stiffness vs. number of cycles (mean values - M, upper - R and lower - L bounds, respectively)

Rys. 1. Degradacja sztywności laminatu (włókna szklane/żywica epoksydowa) w zależności od liczby cykli (wartości średnie - M, kres górny - R, kres dolny - L)

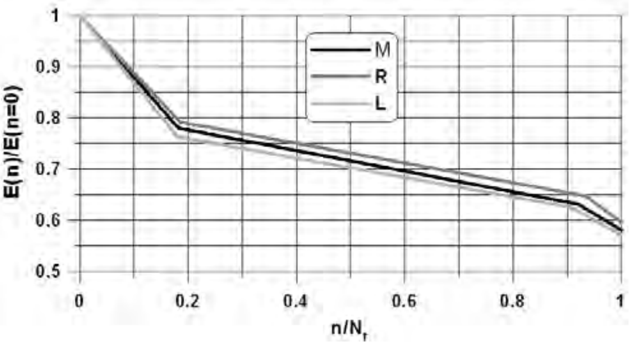

Fig. 2. Degradation of laminate (aramid fiber/epoxy resin) stiffness vs. number of cycles (mean values - M, upper - R and lower - L bounds, respectively)

Rys. 2. Degradacja sztywności laminatu (włókna aramidowe/żywica epoksydowa) w zależności od liczby cykli (wartości średnie - M, kres górny - R, kres dolny - L)

ESTIMATIONS OF FATIGUE LIFE

Let us consider a rectangular plate with a centrally located hole - see Figure 3. The plate is made of glass fibre/epoxy resin and the laminates consist of five layers. The specimens were subjected to cyclic tensile stress.

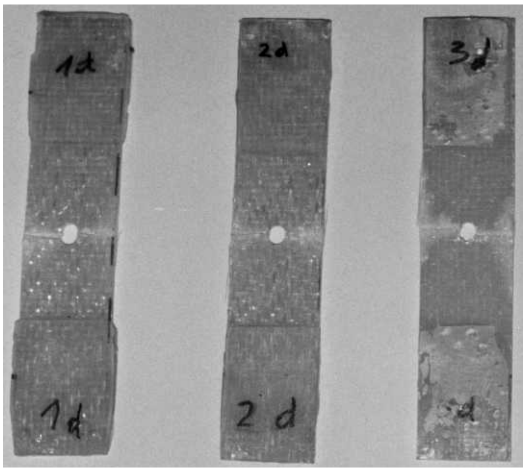

Fig. 3. Failure form of rectangular plate with centrally located hole

Rys. 3. Postać zniszczenia prostokątnych płyt z centralnie umieszczonym otworem

Using the procedures presented above, damage parameter d variations with the number of cycles are plotted in Figure 4. They are also compared with the experimental results. With the use of the methodology of fuzzy logic described in references [8-10], the upper and lower boundaries of parameter *d* are plotted in Figure 4. The scatter of the latter values is even higher than the theoretically established higher boundaries.

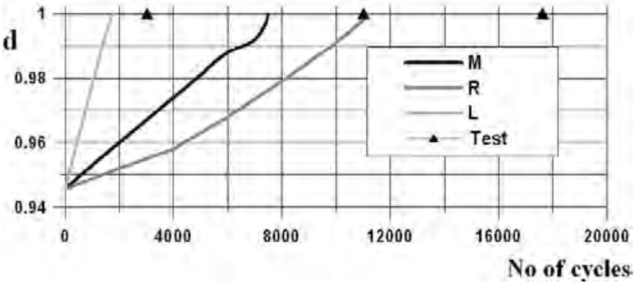

Fig. 4. Theoretical and experimental values of damage parameter *d*

Rys. 4. Teoretyczne i doświadczalne wartości parametru uszkodzeń *d*

The next example deals with the analysis of fatigue behavior and damage of a square plate made of aramid/epoxy resin - see Figure 5. Similarly as previously, the analyzed plate is made of five layers having identical thicknesses.

The damage (Last-Ply-Failure) occurs always in the upper ply. The slow (comparing with the previous example) development of fatigue damage demonstrates the existence of local delaminations and/or fibre debonding. The scatter of the fatigue life is quite high and varies from 16 100 cycles to 45 300 cycles. It is unsymmetric with respect to the mean (deterministic) value denoted by the letter M - see Figure 6. The values of four experimental data lie almost exactly between the upper and lower boundaries. Better agreement can be achieved when the uncertainty of external loads is taken into account since the uniformity of external shearing loads is an assumption only. In addition, the boundary conditions do not reflect exactly the real experimental situation and set-up. However, the agreement between the experiments and theory seems to be reasonably good.

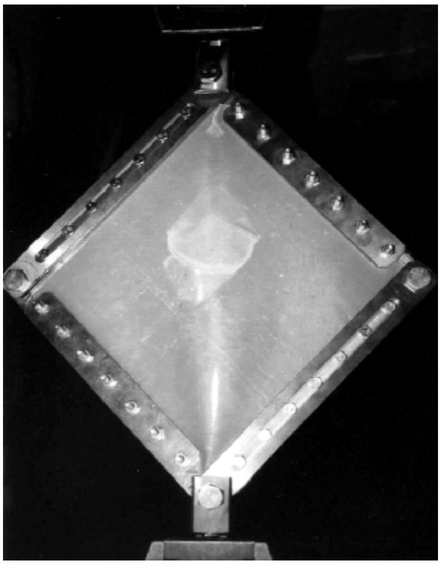

Fig. 5. Photograph of damaged square plate subjected to uniform shear

Rys. 5. Zdjęcie uszkodzonej kwadratowej płyty poddanej równomiernemu ścinaniu

Final fatigue damage is associated with fiber breaking in the fifth ply (Last-Ply-Failure), which is illustrated in Figure 5. It is necessary to add that the observed local buckling of the ply joined with local delamination of the ply is the secondary effect being, in our opinion, the result of the fibre breaking. Usually, local buckling occurs if the delamination area exceeds the critical one - the detailed explanation of this effect can be found e.g. in work [7]. The numerical computations demonstrate that the above condition is not satisfied.

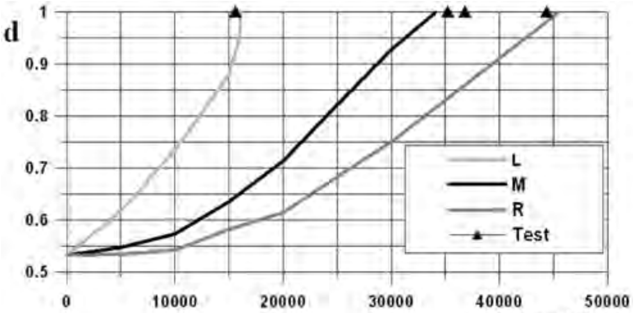

Fig. 6. Theoretical and experimental values of damage parameter *d*

Rys. 6. Teoretyczne i doświadczalne wartości parametru uszkodzeń *d*

CONCLUDING REMARKS

The present study is a practical tool for engineering purposes dealing with the evaluation of fatigue life for real constructions. On the other, the above model of fatigue damage simulation can be easily adopted in optimization problems that concern maximization of fatigue life with respect to laminate configuration (understood in the sense of design variables). The next, still open problem, is connected with the total number of uncertain parameters that should be considered in order to describe with acceptable accuracy the real behavior of engineering structures. However, it can be solved for each individual problem only.

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